Simple Exponential Smoothing

With simple exponential smoothing, the **forecast** is made up of the actual value for the present time period X_t multiplied by a value between 0 and 1 (the exponential smoothing constant) referred to as α (not the same a used for a Type I error) plus the product of the present time period forecast F_t and $(1-\alpha)$. The formula is stated algebraically as follows:

Forecast: A projection or prediction of future values of a time-series.

$$F_{t+1} = \alpha X_t + (1-\alpha) F_t = F_t + \alpha (X_t - F_t) \quad (16\text{-}1a)$$

where F_{t+1} = Forecast for the next time period $(t + 1)$
F_t = forecast for the present time period (t)
α = a weight called exponentially smoothing constant $(0 \le \alpha \le 1)$
X_t = actual value for the present time period (t)

If exponential smoothing has been used over a period of time, the forecast for F_t will have been obtained by

$$F_t = \alpha X_{t-1} + (1-\alpha) F_{t-1}. \quad (16\text{-}1b)$$

When *smoothing constant* α is low, more weight is given to past data, and when it is high, more weight is given to recent data values. When α is equal to 0.9, then 99.99 per cent of the forecast value is determined by the four most recent demands. When α is as low as 0.1, only 34.39 per cent of the average is due to these last 4 periods and the smoothing effect is equivalent to a 19-period arithmetic moving average.

If α were assigned a value as high as 1, each forecast would reflect total adjustment to the recent data value and the forecast would simply be last period's actual value, that is, $F_t = 1.0D_{t-1}$. Since fluctuations are typically random, the value of α is generally kept in the range of 0.005 to 0.30 in order to 'smooth' the forecast.

The following table helps illustrate this concept. For example, when α = 0.5, we can see that the new forecast is based on data value in the last three or four periods. When α = 0.1, the forecast places little weight on recent value and takes a 19-period arithmetic moving average.

Smoothing Constant	*Weight Assigned to*				
	Most Recent Period (α)	*2nd Most Recent Period* $\alpha(1-\alpha)$	*3rd Most Recent Period* $\alpha(1-\alpha)^2$	*4th Most Recent Period* $\alpha(1-\alpha)^3$	*5th Most Recent Period* $\alpha(1-\alpha)^4$
α = 0.1	0.1	0.09	0.081	0.073	0.066
α = 0.5	0.5	0.25	0.125	0.063	0.031

Selecting the smoothing constant The exponential smoothing approach has been successfully applied by banks, manufacturing companies, wholesalers, and other organizations. The appropriate value of the exponential smoothing constant, α, however, can make the difference between an accurate and an inaccurate forecast. In picking a value for the smoothing constant, the objective is to obtain the most accurate forecast.

The correct α-value facilitates a reasonable reaction to a data value without incorporating too much random variation. An approximate value of α which is equivalent to an arithmetic moving average, in terms of degree of smoothing, can be estimated as: $\alpha = 2/(n + 1)$. The accuracy of a forecasting model can be determined by comparing the forecasted values with the actual or observed values.

Error The error of an individual forecast is defined as:

Forecast error = Actual values − Forecasted values

$$e_t = X_t - F_t$$

One measure of the overall forecast error for a model is the *mean absolute deviation (MAD)*. This is computed by taking the sum of the absolute values of the individual forecast errors and then dividing by number of periods n of data

$$\text{MAD} = \frac{\Sigma|\text{ Forecast errors }|}{n}$$

where Standard deviation $\sigma \cong 1.25$ MAD

The exponential smoothing method also facilitates continuous updating of the estimate of MAD. The current MAD_t is given by

$$MAD_t = \alpha \mid \text{Actual values} - \text{Forecasted values} \mid + (1-\alpha)\, MAD_{t-1}$$

Higher values of smoothing constant α make the current MAD more responsive to current forecast errors

Example 16.7: A firm uses simple exponential smoothing with $\alpha = 0.1$ to forecast demand. The forecast for the week of February 1 was 500 units whereas actual demand turned out to be 450 units.

(a) Forecast the demand for the week of February 8.
(b) Assume the actual demand during the week of February 8 turned out to be 505 units. Forecast the demand for the week of February 15. Continue forecasting through March 15, assuming that subsequent demands were actually 516, 488, 467, 554, and 510 units.

Solution: Given $F_{t-1} = 500$, $D_{t-1} = 450$, and $\alpha = 0.1$

(a) $F_t = F_{t-1} + \alpha(D_{t-1} - F_{t-1}) = 500 + 0.1(450 - 500) = 495$ units

(b) Forecast of demand for the week of February 15 is shown in Table 16.5.

Table 16.5 Forecast of Demand

Week	Demand D_{t-1}	Old Forecast F_{t-1}	Forecast Error $(D_{t-1} - F_{t-1})$	Correction $\alpha(D_{t-1} - F_{t-1})$	New Forecast (F_t) $F_{t-1} + \alpha(D_{t-1} - F_{t-1})$
Feb. 1	450	500	−50	−5	495
8	505	495	10	1	496
15	516	496	20	2	498
22	488	498	−10	−1	497
Mar. 1	467	497	−30	−3	494
8	554	494	60	6	500
15	510	500	10	1	501

If no previous forecast value is known, the old forecast starting point may be estimated or taken to be an average of some preceding periods.

Example 16.8: A hospital has used a 9-month moving average forecasting method to predict drug and surgical inventory requirements. The actual demand for one item is shown in the table below. Using the previous moving average data, convert to an exponential smoothing forecast for month 33.

Month	24	25	26	27	28	29	30	31	32
Demand	78	65	90	71	80	101	84	60	73

Solution: The moving average of a 9-month period is given by

$$\text{Moving average} = \frac{\Sigma \text{ Demand } (x)}{\text{Number of periods}} = \frac{78 + 65 + \ldots + 73}{9} = 78$$

Assume $F_{t-1} = 78$. Therefore, estimated $\alpha = \dfrac{2}{n+1} = \dfrac{2}{9+1} = 0.2$

Thus, $\quad F_t = F_{t-1} + \alpha(D_{t-1} - F_{t-1}) = 78 + 0.2(73 - 78) = 77$ units

Adjusted Exponential Smoothing

The simple exponential smoothing models is highly flexible because the smoothing effect can be increased or decreased by lowering or raising the value of α. However, if a trend exists in the data, the series will always lag behind the trend. Thus for an increasing trend the forecasts will be consistently low and for decreasing trends they will be consistently high. Simple exponential smoothing forecasts may be adjusted $(F_t)_{adj}$ for trend effects by adding a trend smoothing factor β to the calculated forecast value F_t.

$$(F_t)_{adj} = F_t + \frac{1-\beta}{\beta} T_t$$

where $(F_t)_{adj}$ = trend-adjusted forecast
F_t = simple exponential smoothing forecast
β = smoothing constant for trend
T_t = exponentially smoothed trend factor

The value of the trend smoothing constant β, resembles the α constant in that a high β is more responsive to recent changes in trend. A low β gives less weight to the most recent trends and tends to smooth out the present trend. Values of β can be found by the trial-and-error approach, with the MAD used as a measure of comparison.

The value of the exponentially smoothed trend factor (T_t) is computed in a manner similar to that used in calculating the original forecast, and may be written as:

$$T_t = \beta(F_t - F_{t-1}) + (1 - \beta)T_{t-1}$$

where T_{t-1} = last period trend factor.

The trend factor T_t consists of a portion (β) of the trend evidenced from the current and previous forecast ($F_t - F_{t-1}$) with the remainder ($1 - \beta$) coming from the previous trend adjustment (T_{t-1}).

Simple exponential smoothing is often referred to as *first-order smoothing* and trend-adjusted smoothing is called *second-order* or *double smoothing*. Other advanced exponential smoothing models are also in use, including seasonal adjusted and triple smoothing.

Example 16.9: Develop an adjusted exponential forecast for the firm in Example 16.7. Assume the initial trend adjustment factor (T_{t-1}) is zero and $\beta = 0.1$.

Solution: Table 16.6 presents information needed to develop an adjusted exponential forecast.

Table 16.6

Week	D_{t-1}	F_{t-1}	F_t
Feb. 1	450	500	495
8	505	495	496
15	516	496	498
22	488	498	497
Mar. 1	467	497	494
8	554	494	500
15	510	500	501

The trend adjustment is an addition of a smoothing factor $\{(1 - \beta)/\beta\}T_t$ to the simple exponential forecast, so we need the previously calculated forecast values. Letting the first $T_{t-1} = 0$, we have

Week 2/1: $T_t = \beta(F_t - F_{t-1}) + (1 - \beta)T_{t-1}$
$= 0.1(495 - 500) + (1 - 0.1)(0) = -0.50$

Adjusted forecast $(F_t)_{adj} = F_t + \frac{1-\beta}{\beta}T_t = 495 + \frac{1-0.1}{0.1}(-0.50) = 490.50$

Week 2/8: $T_t = 0.1(496 - 495) + 0.9(-0.50) = -0.35$

Adjusted forecast $(F_t)_{adj} = 496 + 9(-0.35) = 492.85$

Putting the remainder of the calculations in table form, the trend-adjusted forecast for the week of March 15 is $(F_t)_{adj} = 501.44$ compared to the simple exponential forecast of $F_t = 500$, which is not a large difference.

Self-Practice Problems 16A

16.1 The owner of a small company manufactures a product. Since he started the company, the number of units of the product he has sold is represented by the following time series:

Year :	1995	1996	1997	1998	1999	2000	2001
Units : sold	100	120	95	105	108	102	112

Find the trend line that describes the trend by using the method of semi-averages.

16.2 Fit a trend line to the following data by the freehand method:

Year	*Production of Steel (million tonnes)*	*Year*	*Production of Steel (million tonnes)*
1995	20	2000	25
1996	22	2001	23
1997	24	2002	26
1998	21	2003	25
1999	23		

16.3 A State Govt. is studying the number of traffic fatalities in the state resulting from drunken driving for each of the last 12 months

Month	*Accidents*
1	280
2	300
3	280
4	280
5	270
6	240
7	230
8	230
9	220
10	200
11	210
12	200

Find the trend line that describes the trend by using the method of semi-averages.

16.4 Calculate the three-month moving averages from the following data:

Jan.	Feb.	March	April	May	June
57	65	63	72	69	78
July	Aug.	Sept.	Oct.	Nov.	Dec.
82	81	90	92	95	97

[*Osmania Univ., BCom, 1996*]

16.5 Gross revenue data (Rs in million) for a Travel Agency for a 11-year period is as follows:

Year	*Revenue*
1995	3
1996	6
1997	10
1998	8
1999	7
2000	12
2001	14
2002	14
2003	18
2004	19

Calculate a 3-year moving average for the revenue earned.

16.6 The owner of small manufacturing company has been concerned about the increase in manufacturing costs over the past 10 years. The following data provide a time series of the cost per unit for the company's leading product over the past 10 years.

Year	*Cost per Unit*	*Year*	*Cost per Unit*
1995	332	2000	405
1996	317	2001	410
1997	357	2002	427
1998	392	2003	405
1999	402	2004	438

Calculate a 5-year moving average for the unit cost of the product.

16.7 The following data provide a time series of the number of Commercial and Industrial units failures during the period 1989–2004.

Year	*No. of Failures*	*Year*	*No. of Failures*
1989	23	1997	9
1990	26	1998	13
1991	28	1999	11
1992	32	2000	14
1993	20	2001	12
1994	12	2002	9
1995	12	2003	3
1996	10	2004	1

Calculate a 5-year and 7-year moving average for the number of units failure.

16.8 Estimate the trend values using the data given by taking a four-year moving average :

Year	*Value*	*Year*	*Value*
1990	12	1997	100
1991	25	1998	82
1992	39	1999	65
1993	54	2000	49
1994	70	2001	34
1995	87	2002	20
1996	105	2003	7

[*Madras Univ., MCom, 1998*]

16.9 In January, a city hotel predicted a February demand for 142 room occupancy. Actual February demand was 153 rooms. Using a smoothing constant of $\alpha = 0.20$, forecast the March demand using the exponential smoothing model.

16.10 A shoe manufacturer, using exponential smoothing with $\alpha = 0.1$, has developed a January trend forecast of 400 units for a ladies' shoe. This brand has seasonal indexes of 0.80, 0.90, and 1.20 respectively for the first three months of the year. Assuming actual sales were 344 units in January and 414 units in February, what would be the seasonalized March forecast?

16.11 A food processor uses exponential smoothing (with $\alpha = 0.10$) to forecast next month's demand. Past (actual) demand in units and the simple exponential forecasts up to month 51 are shown in the following table

Month	*Actual Demand*	*Old Forecast*
43	105	100.00
44	106	100.50
45	110	101.05
46	110	101.95
47	114	102.46
48	121	103.61
49	130	105.35
50	128	107.82
51	137	109.84

(a) Using simple exponential smoothing, forecast the demand for month 52.

(b) Suppose a firm wishes to start including a trend-adjustment factor of $\beta = 0.60$. If it assumes an initial trend adjustment of zero ($T_t = 0$) in month 50, what would be the value of $(F_t)_{adj}$ for month 52?

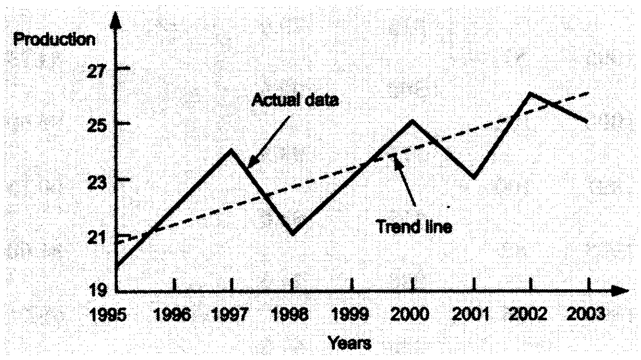
Hints and Answers

16.1

Year (y)	*Units Sold* (x)	
1995	100	
1996	120	} 315/3 = 105.00 = a
1997	95	
1998	105	
1999	108	
2000	102	} 322/3 = 107.33 = b
2001 .	112	

Trend line $y = 105 + 107.33x$.

16.2



16.3

Month	*Accidents*
1	280
2	300
3	280
4	280
5	270
6	240
7	230
8	230
9	220
10	200
11	210
12	200

Average of first 6 months, $a = 1650/6 = 275$

Average of last 6 months, $b = 1290/6 = 215$

Trend line $y = 275 + 215\,x$.

16.4

Month	*Values*	*3-month Total*	*3-month Moving Average*
Jan.	57	—	—
Feb.	65	185	185/3 = 61.67
March	63	200	200/3 = 66.67
April	72	204	204/3 = 68.00
May	69	219	73.00
June	78	229	76.33
July	82	241	80.33
Aug.	81	253	84.33
Sept.	90	263	87.67
Oct.	92	277	92.38
Nov.	95	284	94.67
Dec.	97	—	—

16.5

Year	*Revenue*	*3-year Moving Total*	*3-year Moving Average*
1995	3	—	
1996	6	→ 19	19/3 = 6.33
1997	10	→ 24	24/3 = 8.00
1998	8	→ 21	21/3 = 7.00
1999	7	25	8.33
2000	12	32	10.66
2001	14	34	11.33
2002	14	46	15.33
2003	18	51	17.00
2004	19	—	—

16.6

Year	*Per Unit Cost*	*5-year Moving Total*	*5-year Moving Average*
1995	332	—	—
1996	317	—	—
1997	357	→ 1800	1800/5 = 360.0
1998	392	→ 1873	1873/5 = 374.6
1999	402	1966	1966/5 = 393.2
2000	405	2036	407.2
2001	410	2049	409.8
2002	427	2085	417.0
2003	405	—	—
2004	438	—	—

16.7

Year	*Number of Failures*	*5-year Moving Total*	*5-year Moving Average*	*7-year Moving Total*	*7-year Moving Average*
1989	23	—	—	—	—
1990	26	—	—	—	—
1991	28	→ 129	25.8	—	—
1992	32	→ 118	23.6	153	21.9
1993	20	104	20.8	140	20.0
1994	12	86	17.2	123	17.6
1995	12	63	12.6	108	15.4
1996	10	56	11.2	87	12.4
1997	9	55	11.0	81	11.6
1998	13	57	11.4	81	11.6
1999	11	59	11.8	78	11.1
2000	14	59	11.8	71	10.1
2001	12	69	9.8	63	5.0
2002	9	39	7.9	—	—
2003	3	—	—	—	—
2004	1	—	—	—	—

16.8

Year *Average*	*Value* *Centred*	*4-year*	*4-year*	*4-year Moving* *Total*	*Moving Moving* *Average*
1990	12	—	—	—	—
1991	25	—	—	—	—
		→ 130	130/4=32.5		
1992	39				(32.5 + 47)/2 = 39.75
		→188	188/4=47.0		
1993	54				(47 + 62.5)/2 = 54.75
		→250	250/4=62.5		
1994	70				70.75
		316	79.0		
1995	87				84.75
		362	90.5		
1996	105				92.00
		374	93.5		
1997	100				90.75
		352	88.0		
1998	82				81.00
		296	74.0		
1999	65				65.75
		230	57.5		
2000	49				49.75
		168	42.0		
2001	34				34.75
		110	27.5		
2002	20	—	—		—
2003	7	—	—		—

16.9 New forecast (March demand)

$= F_{t-1} + \alpha (D_{t-1} - F_{t-1})$

$= 142 + 0.20(153 - 142) = 144.20 = 144$ rooms

16.10 (a) Deseasonalized actual January demand

$= 344/0.80 = 430$ units

(b) Compute the deseasonalized forecast

$$F_t = F_{t-1} + \alpha (D_{t-1} - F_{t-1})$$
$$= 400 + 0.1\,(430 - 400) = 403$$

16.11 In this problem the smoothing constant for the original data ($\alpha = 0.10$) differs from the smoothing constant for the trend $\beta = 0.60$.

(*a*) $F_t = F_{t-1} + \alpha (D_{t-1} - F_{t-1})$

$= 109.84 + 0.1(137.00 - 109.84) = 112.56$

(*b*) Forecast for month 51 :

$$(F_t)_{adj} = F_t + \frac{1-\beta}{\beta} T_t$$

where $T_t = \beta (F_t - F_{t-1}) + (1 - \beta)T_{t-1}$

$= 0.6\,(109.84 - 107.82) + (1 - 0.6)\,0$

$= 1.21$

$$(F_t)_{adj} = 109.84 + \left(\frac{1-0.6}{0.6}\right)(1.21)$$
$$= 110.65$$

Forecast for month 52 :

$$T_t = \beta (F_t - F_{t-1}) + (1 - \beta)\,T_{t-1}$$
$$= 0.6\,(112.56 - 109.84) + (1 - 0.6)\,(1.21)$$
$$= 2.12$$

$$(F_t)_{adj} = F_t + \frac{1-\beta}{\beta} T_t$$
$$= 112.56 + \left(\frac{1-0.6}{0.6}\right)(2.12) = 113.98$$

16.9 TREND PROJECTION METHODS

A *trend* is the long-run general direction (upward, downward or constant) of a business climate over a period of several year. It is best represented by a straight line.

The trend projection method fits a trend line to a time series data and then projects medium-to-long-range forecasts. Several possible trend fits can be explored (such as exponential and quadratic), depending upon movement of time-series data. In this section, we will discuss linear, quadratic and exponential trend models. Since seasonal effects can compound trend analysis, it is assumed that no seasonal effects occur in the data or are removed before establishing the trend.

Reasons to study trend: A few reasons to study trends are as follows:

1. The study of trend helps to describe the long-run general direction (upward, downward, constant) of a business climate over a period of several years.
2. The study allows us to use trends as an aid in making intermediate and long-range forecasting projections in the future.
3. The study of trends help to esolate and then eliminate its influencing effects on the time-series model.

16.9.1 Linear Trend Model

The *method of least squares* from regression analysis is used to find the *trend line of best fit* to a time series data. The regression trend line (y) is defined by the following equation.

$$\hat{y} = a + bx$$

where $\hat{y}$ = predicted value of the dependent variable

a = y-axis intercept,

b = slope of the regression line (or the rate of change in y for a given change in x),

x = independent variable (which is *time* in this case).

The trend line of best fit has the properties that (i) the summation of all vertical deviations about it is zero, that is, $\Sigma (y - \hat{y}) = 0$, (ii) the summation of all vertical deviations squared is a minimum, that is, $\Sigma (y - \hat{y})$ is least, and (iii) the line goes through the mean values of variables x and y. For linear equations, it is found by the simultaneous solution for a and b of the two normal equations:

$$\Sigma y = na + b\Sigma x \quad \text{and} \quad \Sigma xy = a\Sigma x + b\Sigma x^2$$

where the data can be coded so that $\Sigma x = 0$, two terms in these equations drop out and we have

$$\Sigma y = na \quad \text{and} \quad \Sigma xy = b\Sigma x^2$$

Coding is easily done with time-series data. For coding the data, we choose the centre of the time period as $x = 0$ and have an equal number of plus and minus periods on each side of the trend line which sum to zero.

Alternately, we can also find the values of constants a and b for any regression line as:

$$b = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n(\bar{x})^2} \quad \text{and} \quad a = \bar{y} - b\bar{x}$$

Example 16.10: Below are given the figures of production (in thousand quintals) of a sugar factory:

Year	:	1995	1996	1997	1998	1999	2000	2001
Production	:	80	90	92	83	94	99	92

(a) Fit a straight line trend to these figures

(b) Plot these figures on a graph and show the trend line.

(c) Estimate the production in 2004. *[Bangalore Univ., BCom, 1998]*

Solution: (a) Using normal equations and the sugar production data we can compute constants a and b as shown in Table 16.7:

Table 16.7 Calculation for Least Squares Equation

Year	*Time Period (x)*	*Production (x)*	x^2	*xy*	*Trend Values* $\hat{y}$
1995	1	80	1	80	84
1996	2	90	4	180	86
1997	3	92	9	276	88
1998	4	83	16	332	90
1999	5	94	25	470	92
2000	6	99	36	594	94
2001	7	92	49	644	96
	28	630	140	2576	

$$\bar{x} = \frac{\Sigma x}{n} = \frac{28}{7} = 4, \quad \bar{y} = \frac{\Sigma y}{n} = \frac{630}{7} = 90$$

$$b = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n(\bar{x})^2} = \frac{2576 - 7(4)(90)}{140 - 7(4)^2} = \frac{56}{28} = 2$$

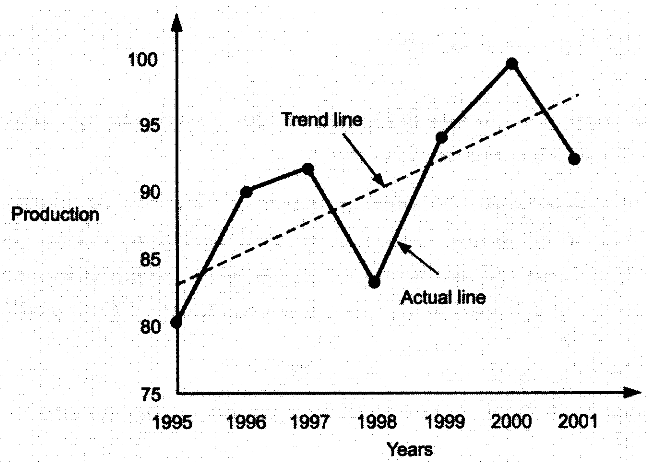
$$a = \bar{y} - b\bar{x} = 90 - 2(4) = 82$$

Therefore, linear trend component for the production of sugar is:

$$\hat{y} = a + bx = 82 + 2x$$

The slope $b = 2$ indicates that over the past 7 years, the production of sugar had an average growth of about 2 thousand quintals per year.

Figure 16.7
Linear Trend for Production of Sugar



(b) Plotting points on the graph paper, we get an actual graph representing production of sugar over the past 7 years. Join the point $a = 82$ and $b = 2$ (corresponds to 1996) on the graph we get a trend line as shown in Fig. 16.7.

(c) The production of sugar for year 2004 will be

$$\hat{y} = 82 + 2\,(10) = 102 \text{ thousand quintals}$$

Example 16.11: The following table relates to the tourist arrivals (in millions) during 1994 to 2000 in India:

Year :	1994	1995	1996	1997	1998	1999	2000
Tourists arrivals :	18	20	23	25	24	28	30

Fit a straight line trend by the method of least squares and estimate the number of tourists that would arrive in the year 2004. *[Kurukshetra Univ., MTM., 1997]*

Solution: Using normal equations and the tourists arrival data we can compute constants a and b as shown in Table 16.8:

Table 16.8 Calculations for Least Squares Equation

Year	Time Scale (x)	Tourist Arrivals y	xy	x^2
1994	−3	18	−54	9
1995	−2	20	−40	4
1996	−1	23	−23	1
1997	0	25	0	0
1998	1	24	24	1
1999	2	28	56	4
2000	3	30	90	9
		168	53	28

$$\bar{x} = \frac{\Sigma x}{n} = 0,\ \bar{y} = \frac{\Sigma y}{n} = \frac{168}{7} = 24$$

$$b = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n(\bar{x})^2} = \frac{53}{28} = 1.893;$$

$$a = \bar{y} - b\bar{x} = 24 - 1.893(0) = 24$$

Therefore, the linear trend component for arrival of tourists is

$$\hat{y} = a + bx = 24 + 1.893x$$

The estimated number of tourists that would arrive in the year 2004 are:

$$\hat{y} = 24 + 1.893\,(7) = 37.251 \text{ million (measured from 1997 = origin)}$$

16.9.2 Quadratic Trend Model

The quadratic relationship for estimating the value of a dependent variable y from an independent variable x might take the form

$$\hat{y} = a + bx + cx^2$$

This trend line is also called the *parabola*.

For a non-linear equation $y = a + bx + cx^2$, the values of constants a, b, and c can be determined by solving three normal equations

$$\Sigma y = na + b\Sigma x + c\Sigma x^2$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$
$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

When the data can be coded so that $\Sigma x = 0$ and $\Sigma x^3 = 0$, two terms in the above expressions drop out and we have

$$\Sigma y = na + c\Sigma x^2$$
$$\Sigma xy = b\Sigma x^2$$
$$\Sigma x^2 y = a\Sigma x^2 + c\Sigma x^4$$

To find the exact estimated value of the variable y, the values of constants a, b, and c need to be calculated. The values of these constants can be calculated by using the following shortest method:

$$a = \frac{\Sigma y - c\Sigma x^2}{n};\quad b = \frac{\Sigma xy}{\Sigma x^2} \text{ and } c = \frac{n\Sigma x^2y - \Sigma x^2 \Sigma y}{n\Sigma x^4 - (\Sigma x^2)^2}$$

Example 16.12: The prices of a commodity during 1998–2003 are given below. Fit a parabola to these data. Estimate the price of the commodity for the year 2004.

Year	*Price*	*Year*	*Price*
1998	100	2001	140
1999	107	2002	181
2000	128	2003	192

Also plot the actual and trend values on a graph.

Solution: To fit a quadratic equation $\hat{y} = a + bx + cx^2$, the calculations to determine the values of constants a, b, and c are shown in Table 16.9.

Table 16.9 Calculations for Parabola Trend Line

Year	*Time Scale (x)*	*Price (y)*	x^2	x^3	x^4	xy	x^2y	*Trend Values* ($\hat{y}$)
1998	−2	100	4	−8	16	−200	400	97.72
1999	−1	107	1	−1	1	−107	107	110.34
2000	0	128	0	0	0	0	0	126.68
2001	1	140	1	1	1	140	140	146.50
2002	2	181	4	8	16	362	724	169.88
2003	3	192	9	27	81	576	1728	196.82
	3	848	19	27	115	771	3099	847.94

(i) $\Sigma y = na + b\Sigma x + c\Sigma x^2$ or $848 = 6a + 3b + 19c$

(ii) $\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$ or $771 = 3a + 19b + 27c$

(iii) $\Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$ or $3099 = 19a + 27b + 115c$

Eliminating a from eqns. (i) and (ii), we get

(iv) $694 = 35b + 35c$

Eliminating a from eqns. (ii) and (iii), we get

(v) $5352 = 280b + 168c$

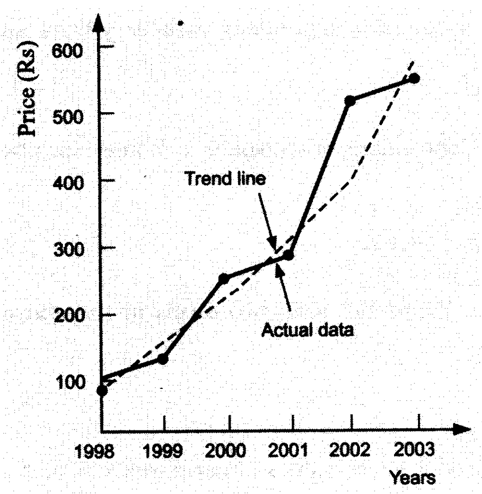
Solving eqns. (iv) and (v) for b and c we get $b = 18.04$ and $c = 1.78$. Substituting values of b and c in eqn. (i), we get $a = 126.68$.

Hence, the required non-linear trend line becomes

$$y = 126.68 + 18.04x + 1.78x^2$$

Several trend values as shown in Table 16.9 can be obtained by putting $x = -2, -1, 0, 1, 2$, and 3 in the trend line. The trend values are plotted on a graph paper. The graph is shown in Fig. 16.8.

Figure 16.8
Trend Line for Price of Commodity



16.9.3 Exponential Trend Model

When the given values of dependent variable y form approximately a geometric progression while the corresponding independent variable x values form an arithmetic progression, the relationship between variables x and y is given by an exponential function, and the best fitting curve is said to describe the *exponential trend*. Data from the fields of biology, banking, and economics frequently exhibit such a trend. For example, growth of bacteria, money accumulating at compound interest, sales or earnings over a short period, and so on, follow exponential growth.

The characteristic property of this law is that the rate of growth, that is, the rate of change of y with respect to x is proportional to the values of the function. The following function has this property.

$$y = a\, b^{cx}, a > 0$$

The letter b is a fixed constant, usually either 10 or e, where a is a constant to be determined from the data.

To assume that the law of growth will continue is usually unwarranted, so only short range predictions can be made with any considerable degree of reliability.

If we take logarithms (with base 10) of both sides of the above equation, we obtain

$$\log y = \log a + (c \log b)\, x$$

For $b = 10$, $\log b = 1$, but for $b = e$, $\log b = 0.4343$ (approx.). In either case, this equation is of the form

$$y' = c + dx \qquad (16\text{-}2)$$

where $y' = \log y$, $c = \log a$, and $d = c \log b$.

Equation (16-2) represents a straight line. A method of fitting an exponential trend line to a set of observed values of y is to fit a straight trend line to the logarithms of the y-values.

In order to find out the values of constants a and b in the exponential function, the two normal equations to be solved are

$$\Sigma \log y = n \log a + \log b\, \Sigma x$$

$$\Sigma x \log y = \log a\, \Sigma x + \log b\, \Sigma x^2$$

When the data is coded so that $\Sigma x = 0$, the two normal equations become

$$\Sigma \log y = n \log a \quad \text{or} \quad \log a = \frac{1}{n} \Sigma \log y$$

and

$$\Sigma x \log y = \log b\, \Sigma x^2 \quad \text{or} \quad \log b = \frac{\Sigma x \log y}{\Sigma x^2}$$

Coding is easily done with time-series data by simply designating the center of the time period as $x = 0$, and have equal number of plus and minus period on each side which sum to zero.

Example 16.13: The sales (Rs in million) of a company for the years 1995 to 1999 are:

Year :	1997	1998	1999	2000	2001
Sales :	1.6	4.5	13.8	40.2	125.0

Find the exponential trend for the given data and estimate the sales for 2004.

Solution: The computational time can be reduced by coding the data. For this consider $u = x - 3$. The necessary computations are shown in Table 16.10.

Table 16.10 Calculation for Least Squares Equation

Year	Time Period x	$u = x - 3$	u^2	Sales y	Log y	u Log y
1997	1	−2	4	1.60	0.2041	−0.4082
1998	2	−1	1	4.50	0.6532	−0.6532
1999	3	0	0	13.80	1.1390	0
2000	4	1	1	40.20	1.6042	1.6042
2001	5	2	4	125.00	2.0969	4.1938
			10		5.6983	4.7366

$$\log a = \frac{1}{n}\,\Sigma \log y = \frac{1}{5}\,(5.6983) = 1.1397$$

$$\log b = \frac{\Sigma u \log y}{\Sigma u^2} = \frac{4.7366}{10} = 0.4737$$

Therefore $\log y = \log a + (x + 3) \log b = 1.1397 + 0.4737x$

For sales during 2004, $x = 3$, and we obtain

$$\log y = 1.1397 + 0.4737\,(3) = 2.5608$$

or

$$y = \text{antilog}\,(2.5608) = 363.80$$

16.9.4 Changing the Origin and Scale of Equations

When a moving average or trend value is calculated it is assumed to be centred in the middle of the month (fifteenth day) or the year (July 1). Similarly, the forecast value is assumed to be centred in the middle of the future period. However, the reference point (origin) can be shifted, or the units of variables x and y are changed to monthly or quarterly values if desired. The procedure is as follows:

(i) Shift the origin, simply by adding or subtracting the desired number of periods from independent variable x in the original forecasting equation.

(ii) Change the time units from annual values to monthly values by dividing independent variable x by 12.

(iii) Change the y units from annual to monthly values, the entire right-hand side of the equation must be divided by 12.

Example 16.14: The following forecasting equation has been derived by a least-squares method:

$$\hat{y} = 10.27 + 1.65x \quad \text{(Base year: 1997;} \quad x = \text{years;} \quad y = \text{tonnes/year)}$$

Rewrite the equation by

(a) shifting the origin to 2002.

(b) expressing x units in months, retaining y in tonnes/year.

(c) expressing x units in months and y in tonnes/month.

Solution: (a) Shifting of origin can be done by adding the desired number of period 5 (1997 to 2002) to x in the given equation. That is

$$\hat{y} = 10.27 + 1.65\,(x + 5) = 18.52 + 1.65x$$

where $2002 = 0, x = \text{years}, y = \text{tonnes/year}$.

(b) Expressing x units in months

$$\hat{y} = 10.27 + \frac{1.65x}{12} = 10.27 + 0.14x$$

where July 1, $1997 = 0, x = \text{months}, y = \text{tonnes/year}$.

(c) Expressing y in tonnes/month, retaining x in months

$$\hat{y} = \frac{1}{12}(10.27 + 0.14x) = 0.86 + 0.01x$$

where July 1, $1997 = 0, x = \text{months}, y = \text{tonnes/month}$.

Remarks

1. If both x and y are to be expressed in months together, then divide constant 'a' by 12 and constant 'b' by 24. It is because data are sums of 12 months. Thus monthly trend equation becomes

Linear trend : $\hat{y} = \dfrac{a}{12} + \dfrac{b}{24}x$

Parabolic trend : $\hat{y} = \dfrac{a}{12} + \dfrac{b}{144}x + \dfrac{c}{1728}x^2$

But if data are given as monthly averages per year, then value of 'a' remains unchanged, 'b' is divided by 12 and 'c' by 144.

2. The annual trend equation can be reduced to quarterly trend equation as:

$$\hat{y} = \frac{a}{4} + \frac{b}{4 \times 12}x = \frac{a}{4} + \frac{b}{48}x$$

Self-Practice Problems 16B

16.12 The general manager of a building materials production plant feels that the demand for plasterboard shipments may be related to the number of construction permits issued in the country during the previous quarter. The manager has collected the data shown in the table.

Construction Permits	*Plasterboard Shipments*
15	6
9	4
40	16
20	6
25	13
25	9
15	10
35	16

(a) Use the normal equations to derive a regression forecasting equation.

(b) Determine a point estimate for plasterboard shipments when the number of construction permits is 30.

16.13 A company that manufactures steel observed the production of steel (in metric tonnes) represented by the time-series:

Year	1996	1997	1998	1999	2000	2001	2002
Production of steel	60	72	75	65	80	85	95

(a) Find the linear equation that describes the trend in the production of steel by the company.

(b) Estimate the production of steel in 2003.

16.14 Fit a straight line trend by the method of least squares to the following data. Assuming that the same rate of change continues, what would be the predicted earning (Rs in lakh) for the year 2004?

Year	1995	1996	1997	1998	1999	2000	2001	2002
Earnings	38	40	65	72	69	60	87	95

[*Agra Univ., BCom 1996; MD Univ., BCom, 1998*]

16.15 The sales (Rs in lakh) of a company for the years 1990 to 1996 are given below:

Year	1998	1999	2000	2001	2002	2003	2004
Sales	32	47	65	88	132	190	275

Find trend values by using the equation $y_c = a\,b^x$ and estimate the value for 2005.

[*Delhi Univ., BCom, 1996*]

16.16 A company that specializes in the production of petrol filters has recorded the following production (in 1000 units) over the last 7 years.

Years	1995	96	97	98	99	00	01
Production	42	49	62	75	92	122	158

(a) Develop a second-degree estimating equation that best describes these data.

(b) Estimate the production in 2005.

16.17 In 1996 a firm began downsizing in order to reduce its costs. One of the results of these cost cutting measures has been a decline in the percentage of private industry jobs that are managerial. The following data show the percentage of females who are managers from 1996 to 2003.

Years	1996	97	98	99	00	01	02	03
Percentage	6.7	5.3	4.3	6.1	5.6	7.9	5.8	6.1

(a) Develop a linear trend line for this time series through 2001 only.

(b) Use this trend to estimate the percentage of females who are managers in 2004.

16.18 A company develops, markets, manufactures, and sells integrated wide-area network access products. The following are annual sales (Rs in million) data from 1998 to 2004.

Year	1998	1999	2000	2001	2002	2003	2004
Sales	16	17	25	28	32	43	50

(a) Develop the second-degree estimating equation that best describes these data.

(b) Use the trend equation to forecast sales for 2005.

Hints and Answers

16.12 (a)

x	y	xy	x^2	y^2
15	6	90	225	36
9	4	36	81	16
40	16	640	1,600	256
20	6	120	400	36
25	13	325	625	169
25	9	225	625	81
15	10	150	225	100
35	16	560	1,225	256
184	80	2,146	5,006	950

n= 8 pairs of observations;

$\bar{x}$ = 184/8 = 23; $\bar{y}$ = 80/8 = 10

$\Sigma y = na + b\Sigma x$ or $80 = 8a + 184b$

$\Sigma xy = \Sigma x + b\Sigma x^2$ or $2{,}146 = 184a + 5{,}006b$

After solving equations we get $a = 0.91$ and $b = 0.395$.

Therefore the equation is: $\hat{y} = 0.91 + 0.395x$

(b) For $x = 30$, we have $\hat{y} = 0.91 + 0.395\ (30) = 13$ shipments (approx.)

16.13 $a = \Sigma y/n = 532/7 = 76$; $b = \Sigma xy/\Sigma x^2 = 136/28 = 4.857$

(a) Trend line $\hat{y} = a + bx = 76 + 4.857x$

(b) For 2003, $x = 4$, $\hat{y} = 76 + 4.857(4) = 95.428$ metric tonnes.

16.14 $a = \Sigma y/n = 526/8 = 65.75$;

$b = \Sigma xy/\Sigma x^2 = 616/168 = 3.667$

Trend line : $\hat{y} = a + bx = 65.75 + 3.667x$

For 2004, $x = 11$; $\hat{y} = 65.75 + 3.667\ (11)$

= Rs 106.087 lakh.

16.15 $\log a = \frac{1}{n}\ \Sigma \log y = \frac{1}{7}\ (13.7926) = 1.9704$

$\log b = \frac{\Sigma x \log y}{\Sigma x^2} = \frac{4.3237}{28} = 0.154$

Thus $\log y = \log a + x \log b = 1.9704 + 0.154x$

For 2005, $x = 4$; $\log y = 1.9704 + 0.154\ (4)$

= 2.5864

y = Antilog (2.5864) = Rs 385.9 lakh.

16.16

Year	Period	Deviation from 1998 (x)	x^2	x^4	y	xy	x^2y
1995	1	− 3	9	81	42	− 126	378
1996	2	− 2	4	16	49	− 98	196
1997	3	− 1	1	1	62	− 62	62
1998	4	0	0	0	75	0	0
1999	5	1	1	1	92	+ 92	92
2000	6	2	4	16	122	+ 244	488
2001	7	3	9	81	158	+ 474	1422
		0	28	196	600	524	2638

(a) Solving the equations

$\Sigma y = na + c\Sigma x^2$ or $600 = 7a + 28c$

$\Sigma x^2y = a\Sigma x^2 + c\Sigma x^4$ or $2638 = 28a + 196c$

$\Sigma xy = b\Sigma x^2$ or $524 = 28b$

We get $a = 80.05$, $b = 18.71$ and $c = -1.417$

Hence $\hat{y} = a + bx + cx^2 = 80.05 + 18.71x - 1.417x^2$

(b) For 2005, $x = 8$; $\hat{y} = 80.05 + 18.71(8) - 1.417(8)^2$

= Rs 139.042 thousand.

16.17

Year	Time Period	Deviation from 2001 x	Percentage of Females y	xy	x^2
1996	1	− 5	6.7	− 33.5	25
1997	2	− 4	5.3	− 21.2	16
1998	3	− 3	4.3	− 12.9	9
1999	4	− 2	6.1	− 12.2	4
2000	5	− 1	5.6	− 6.6	1
2001	6	0	7.9	0	0
2002	7	1	5.8	5.8	1
2003	8	2	6.1	12.2	4
		− 12	47.8	−68.4	60

(a) Solving the equations

$\Sigma y = na + b\Sigma x$ or $47.8 = 8a - 12b$

$\Sigma xy = a\Sigma x + b\Sigma x^2$ or $-67.4 = -12a + 60b$

We get $a = 6.28$ and $b = 0.102$

Hence $\hat{y} = a + bx = 6.128 + 0.102x$

(b) For 2004, $x = 3$; $\hat{y} = 6.128 + 0.102\ (3)$

= 6.434 per cent.

16.18

Year	Time Period	Deviation from 2001 (x)	Sales y	xy	x^2	x^4	x^2y
1998	1	− 3	16	− 48	9	81	144
1999	2	− 2	17	− 34	4	16	68
2000	3	− 1	25	− 25	1	1	25
2001	4	0	28	0	0	0	0
2002	5	1	32	32	1	1	32
2003	6	2	43	86	4	16	172
2004	7	3	50	150	9	81	450
		0	211	161	28	196	891

(a) Solving the equations

$\Sigma y = na + c\Sigma x^2$ or $211 = 7a + 28c$

$\Sigma x^2y = a\Sigma x^2 + c\Sigma x^4$ or $891 = 28a + 196c$

$\Sigma xy = b\Sigma x^2$ or $161 = 28b$

We get $a = 27.904$, $b = 5.75$ and $c = 0.559$

$\hat{y} = a + bx + cx^2 = 27.904 + 5.75x + 0.559x^2$

For 2005, $x = 4$; $\hat{y} = 27.904 + 5.75(4) + 0.559\ (4)^2$

= 59.848

16.10 MEASUREMENT OF SEASONAL EFFECTS

As mentioned earlier that time-series data consists of four components: trend, cyclical effects, seasonal effects and irregular fluctuations. In this section, we will discuss techniques for identifying seasonal effects in a time-series data. Seasonal effect is defined as the repetitive and predictable pattern of data behaviour in a time-series around the trend line during particular time intervals of the year. In order to measure (or detect) the seasonal effect, time period must be less than one year such as days, weeks, months, or quarters.

Seasonal effects arises as the result of natural changes in the seasons during the year or may result due to habits, customs, or festivals that occur at the same time year after year.

We have three main reasons to study seasonal effects:

(i) The description of the seasonal effect provides a better understanding of the impact this component has upon a particular time-series.

(ii) Once the seasonal pattern that exists is established, seasonal effect can be climinated from the time-series in order to observe the effect of the other components, such as cyclical and irregular components. Elimination of seasonal effect from the series is referred to as **deseasonalizing** or ***seasonal adjusting*** of the data.

(iii) Trend analysis may be adequate for long-range forecast, but for short-run predictions, knowledge of seasonal effects on time-series data is essential for projection of past pattern into the future.

Deseasonalization: A statistical process used to remove the effect of seasonality from a time-series by dividing each original series observation by the corresponding seasonal index.

Remarks:

1. In an additive time-series model, we can estimate the seasonal component as:

$$S = Y - (T + C + I)$$

In the absence of C and I, we have S = Y – T. That is, the seasonal component is the difference between actual data values in series and the trend values.

2. One of the technique for isolating the effects of seasonality is decomposition. The process of decomposition begins by determining T.C for each and dividing the time-series data (T.C.S.I) by T.C. The resulting expression contains seasonal effects along with irregular fluctuations

$$\frac{T.C.S.I}{T.C} = S.I.$$

A method for diminating irregular fluctuations can be applied, leaving only the seasonal effects as shown below.

$$\text{Seasonal effect} = \frac{T.S.C.I}{T.C.I} = \frac{Y}{T.C.I} \times 100\%$$

3. The process of eliminating the effects of seasonality from a time-series data is referred to as *de-seasonalization* or *seasonal adjustment*. The data can be deseasonalized by dividing the actual values Y by final adjusted seasonal effects, and is expressed as:

$$\frac{Y}{S} = \frac{T.S.C.I}{S} = T.\ C.\ I \times 100\% \quad \leftarrow \text{Multiplicative}$$

$$Y - S = (T + S + C + I) - S = T + C + I \quad \leftarrow \text{Additive Model}$$

Each adjusted seasonal index measures the average magnitude of seasonal influence on the actual values of the time series for a given period within a year. By subtracting the base index of 100 (which represents the T and C components) from each seasonal index, the extent of the influence of seasonal force can be measured.

16.10.1 Seasonal Index

Seasonal effects are measured in terms of an index, called *seasonal index,* attached to each period of the time series within a year. Hence, if monthly data are considered, there are 12 separate seasonal indexes, one for each month. Similarly for quarterly data, there are 4 separate indexes. *A seasonal index is an average that indicates the percentage deviation of actual values of the time series from a base value which excludes the short-term seasonal influences.* The base time series value represents the trend/cyclical influences only.

The following four methods are used to construct seasonal indexes to measure seasonal effects in the time-series data:

(i) Method of simple averages
(ii) Ratio-to-trend method
(iii) Ratio-to-moving average method
(iv) Link relatives method.

16.10.2 Method of Simple Averages

This method is also called ***average percentage method*** because this method expresses the data of each month or quarter as a percentage of the average of the year. The steps of the method are summarized below:

(i) Average the unadjusted data by years and months (or quarters if quarterly data are given).
(ii) Add the figures of each month and obtain the averages by dividing the monthly totals by the number of years. Let the averages for 12 months be denoted by $\bar{x}_1$, $\bar{x}_2$, ..., $\bar{x}_{12}$.
(iii) Obtain an average of monthly averages by dividing the total of monthly averages by 12. That is

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \ldots + \bar{x}_{12}}{12}$$

(iv) Compute seasonal indexes for different months by expressing monthly averages as percentages of the grand average $\bar{\bar{x}}$ as follows:

$$\text{Seasonal index for month } i = \frac{\text{Monthly average for month } i}{\text{Average of monthly averages}} \times 100$$

$$= \frac{\bar{x}_i}{\bar{\bar{x}}} \times 100 \; (i = 1, 2, \ldots, 12)$$

It is important to note that the average of the indexes will always be 100, that is, sum of the indexes should be 1200 for 12 months, and sum should be 400 for 4 quarterly data. If the sum of these 12 months percentages is not 1200, then the monthly percentage so obtained are adjusted by multiplying these by a suitable factor [1200 ÷ (sum of the 12 values)].

Example 16.15: The seasonal indexes of the sale of readymade garments in a store are given below:

Quarter	*Seasonal Index*
January to March	98
April to June	90
July to September	82
October to December	130

If the total sales of garments in the first quarter is worth Rs 1,00,000, determine how much worth of garments of this type should be kept in stock to meet the demand in each of the remaining quarters. *[Delhi Univ., BCom, 1996]*

Solution: Calculations of seasonal index for each quarter and estimated stock (in Rs) is shown in Table 16.11

Table 16.11 Calculation of Estimated Stock

Quarter	*Seasonal Index (SI)*	*Estimated Stock (Rs)*
Jan. —March	98	1,00,000.00
April—June	90	91,836.73*
July —Sept.	82	83,673.45
Oct. —Dec.	130	1,32,653.06

* These figures are calculated as follows:

$$\text{Seasonal index for second quarter} = \frac{\text{Figure for first quarter} \times \text{SI for second quarter}}{\text{SI for first quarter}}$$

$$\text{Seasonal index for third quarter} = \frac{\text{Figure for first quarter} \times \text{SI for third quarter}}{\text{SI for first quarter}}$$

Example 16.16: Use the method of monthly averages to determine the monthly indexes for the data of production of a commodity for the years 2002 to 2004.

Month	*2002*	*2003*	*2004*
January	15	23	25
February	16	22	25
March	18	28	35
April	18	27	36
May	23	31	36
June	23	28	30
July	20	22	30
August	28	28	34
Septermber	29	32	38
October	33	37	47
November	33	34	41
December	38	44	53

Solution: Computation of seasonal index by average percentage method based on the data is shown in Table 16.12.

Table 16.12 Calculation of Seasonal Indexes

Month	*2002*	*2003*	*2004*	*Monthly Total for 3 Years*	*Monthly Averages for 3 Years*	*Percentage Average of Monthly Averages*
Jan.	15	23	25	63	21	70
Feb.	16	22	25	63	21	70
March	18	28	35	81	27	90
April	18	27	36	81	27	90
May	23	31	36	90	30	100
June	23	28	30	81	27	90
July	20	22	30	72	24	80
Aug.	28	28	34	90	30	100
Sept.	29	32	38	99	33	110
Oct.	33	37	47	117	39	130
Nov.	33	34	41	108	36	120
Dec.	38	44	53	135	45	150
				1080	360	1200

Monthly Average : 1080/20 = 90; 360/12 = 30; 1200/2 = 100

The average of monthly averages is obtained by dividing the total of monthly averages by 12. In column 7 each monthly average for 3 years have been expressed as a percentage of the averages. For example, the percentage for January is:

Monthly index for January = 21/30 = 70;
February = (21/30) × 100 = 70
March = (27/30) × 100 = 90, and so on

Example 16.17: The data on prices (Rs in per kg) of a certain commodity during 2000 to 2004 are shown below:

Quarter	*Years*				
	2000	*2001*	*2002*	*2003*	*2004*
I	45	48	49	52	60
II	54	56	63	65	70
III	72	63	70	75	84
IV	60	56	65	72	66

Compute the seasonal indexes by the average percentage method and obtain the deseasonalized values.

Solution: Calculations for quarterly averages are shown in Table 16.13.

Table 16.13 Calculation Seasonal Indexes

Year	*Quarters*			
	I	*II*	*III*	*IV*
2000	45	54	72	60
2001	48	56	63	56
2002	49	63	70	65
2003	52	65	75	72
2004	60	70	84	66
Quarterly total	254	308	364	319
Quarterly average	50.8	61.6	72.8	63.8
Seasonal index	81.60	98.95	116.94	102.48

$$\text{Average of quarterly averages} = \frac{50.8 + 61.6 + 72.8 + 63.8}{4} = \frac{249}{4} = 62.25$$

Thus,

$$\text{Seasonal index for quarter I} = \frac{50.8}{62.25} \times 100 = 81.60$$

$$\text{Seasonal index for quarter II} = \frac{61.6}{62.25} \times 100 = 98.95$$

$$\text{Seasonal index for quarter III} = \frac{72.8}{62.25} \times 100 = 116.94$$

$$\text{Seasonal index for quarter IV} = \frac{63.8}{62.25} \times 100 = 102.48$$

Deseasonalized Values Seasonal in fluences are removed from a time-series data by dividing the actual y value for each quarter by its corresponding seasonal index:

$$\text{Deseasonalized value} = \frac{\text{Actual quarterly value}}{\text{Seasonal index of corresponding quarter}} \times 100$$

The deseasonalized y values which are measured in the same unit as the actual values, reflect the collective influence of *trend, cyclical* and *irregular* forces. The deseasonalized values are given in Table 16.14.

Table 16.7 Calculation for Least Squares Equation

Year	*Quarters*			
	I	*II*	*III*	*IV*
2000	55.14	54.57	61.57	58.54
2001	58.82	56.59	53.87	54.64
2002	60.00	63.66	59.85	63.42
2003	63.72	65.68	64.13	70.25
2004	73.52	70.74	71.83	64.40

Limitations of the method of simple averages This method is the simplest of all the methods for measuring seasonal variation. However, the limitation of this method is that it assumes that there is no trend component in the series, that is, $C \cdot S \cdot I = 0$ or trend is assumed to have little impact on the time-series. This assumption is not always justified.

16.10.3 Ratio-to-Trend Method

This method is also known as the ***percentage trend method.*** This method is an improvement over the method of simple averages. Because here it is assumed that seasonal variation for a given month is a constant fraction of trend. The ratio-to-trend method isolates the seasonal factor when the following ratios are computed:

$$\frac{T \cdot S \cdot C \cdot I}{T} = S \cdot C \cdot I$$

The steps of the method are summarized as follows:

(i) Compute the trend values by applying the least-squares method.

(ii) Eliminate the trend value. In a multiplicative model the trend is eliminated by dividing the original data values by the corresponding trend values and multiplying these ratios by 100. The values so obtained are free from trend.

(iii) Arrange the percentage data values obtained in Step (ii) according to months or quarters as the case may be for the various years.

(iv) Find the monthly (or quarterly) averages of figures arranged in Step (iii) with any one of the usual measures of central tendency—arithmetic mean, median.

(v) Find the grand average of monthly averages found in Step (iv). If the grand average is 100, then the monthly averages represent seasonal indexes. Otherwise, an adjustment is made by multiplying each index by a suitable factor [1200/(sum of the 12 values)] to get the final seasonal indexes.

Example 16.18: Quarterly sales data (Rs in million) in a super bazar are presented in the following table for a four-year period

Year	*Quarters*			
	I	*II*	*III*	*IV*
2000	60	80	72	68
2001	68	104	100	88
2002	80	116	108	96
2003	108	152	136	124
2004	160	184	172	164

Calculate the seasonal index for each of the four quarters using the ratio-to-trend method.

Solution: Calculations to obtain annual trend values from the given quarterly data using the method of least-squares are shown in Table 16.15.

Table 16.15 Calculation of Trend Values

Year	Yearly Total (1)	Yearly Average y = (2)/4	Deviation From Mid-Year x	x^2	xy	Trend Values $\hat{y}$
2000	280	70	-2	4	-140	64
2001	360	90	-1	1	-90	88
2002	400	100	0	0	0	0
2003	520	130	1	1	130	112
2004	680	170	2	4	340	160
		560		10	240	

Solving the following normal equations, we get

$\Sigma y = na + b\Sigma x$ $\quad 560 = 5a$ or $a = 112$

$\Sigma xy = a\Sigma x + b\Sigma x^2$ $\quad 240 = 10b$ or $b = 24$

Thus the yearly fitted trend line is: $y = 112 + 24x$. The value of $b = 24$ indicates yearly increase in sales. Thus the quarterly increment will be $24/4 = 6$.

To calculate quarterly trend values, consider first the year 2000. The trend value for this year is 64. This is the value for the middle of the year 2000, that is, half of the 2nd quarter and half of the 3rd quarter. Since quarterly increment is 6, the trend value for the 2nd quarter of 2000 would be $64 - (6/2) = 61$ and for the 3rd quarter it would be $64 + (6/2) = 67$. The value for the 1st quarter of 2000 would be $61 - 6 = 55$ and for the 4th quarter it would be $67 + 6 = 73$. Similarly, trend values of the various quarters of other years can be calculated as shown in Table 16.16.

Table 16.16 Quarterly Trend Values

Year	Quarters			
	I	II	III	IV
2000	55	61	67	73
2001	79	85	91	97
2002	103	109	115	121
2003	127	133	139	145
2004	151	157	163	169

After getting the trend values, the given data values in the time-series are expressed as percentages of the corresponding trend values in Table 16.16. Thus for the 1st quarter of 2000, this percentage would be $(60/55) \times 100 = 109.09$; for the 2nd quarter it would be $(80/61) \times 100 = 131.15$, and so on. Other values can be calculated in the same manner as shown in Table 16.17.

Table 16.17 Ratio-to-Trend Values

Year	Quarters			
	I	II	III	IV
2000	109.09	131.15	107.46	93.15
2001	86.08	122.35	109.89	90.72
2002	77.67	106.42	93.91	79.34
2003	85.04	114.29	97.84	85.52
2004	105.96	117.20	105.52	97.04
Total	463.84	591.41	514.62	445.77
Average	92.77	118.28	102.92	89.15
				= 403.12
Adjusted seasonal index	92.02	117.33	102.09	88.43

The total of average of seasonal indexes is 403.12 (>400). Thus we apply the correction factor (400/403.12) = 0.992. Now each quarterly average is multiplied by 0.992 to get the adjusted seasonal index as shown in Table 16.17.

The seasonal index 92.02 in the first quarter means that on average sales trend to be depressed by the presence of seasonal forces to the extent of approx. (100 – 92.02) = 7.98%. Alternatively, values of time series would be approx. (7.98/92.02)×100 = 8.67% higher had seasonal influences not been present.

16.10.4 Ratio-to-Moving Average Method

This method is also called the ***percentage moving average method***. In this method, the original values in the time-series data are expressed as percentages of moving averages instead of percentages of trend values in the ratio-to-trend method. The steps of the method are summarized as follows:

(i) Find the centred 12 monthly (or 4 quarterly) moving averages of the original data values in the time-series.

(ii) Express each original data value of the time-series as a percentage of the corresponding centred moving average values obtained in Step (i). In other words, in a multiplicative time-series model, we get

$$\frac{\text{Original data values}}{\text{Trend values}} \times 100 = \frac{T \cdot C \cdot S \cdot I}{T \cdot C} \times 100 = (S \cdot I) \times 100\%$$

This implies that the ratio-to-moving average represents the seasonal and irregular components.

(iii) Arrange these percentages according to months or quarter of given years. Find the averages over all months or quarters of the given years.

(iv) If the sum of these indexes is not 1200 (or 400 for quarterly figures), multiply them by a correction factor = 1200/(sum of monthly indexes). Otherwise, the 12 monthly averages will be considered as seasonal indexes.

Example 16.19: Calculate the seasonal index by the ratio-to-moving method from the following data:

Year	*Quarters*			
	I	*II*	*III*	*IV*
2001	75	60	53	59
2002	86	65	63	80
2003	90	72	66	85
2004	100	78	72	93

Solution: Calculations for 4 quarterly moving averages and ratio-to-moving averages are shown in Table 16.18.

Table 16.18 Calculation of Ratio-to-Moving Averages

Year	*Quarter*	*Original Values* $Y = T.C.S.I$	*4-Quarter Moving Total*	*4-Quarter Moving Average*	*2× 4-Quarter Moving Average* $T.C$	*Ratio-to-Moving Average (Percent)* $\frac{Y}{T.C} = (S.I)100\%$
2001	1	75	—	—	—	—
	2	60	248		—	—
	3	54	259	507	63.375	54/63.375 = 85.20
	4	59	264	523	65.375	59/65.375 = 90.25
2002	1	86	273	537	67.125	128.12
	2	65	294	567	70.875	91.71
	3	62	298	592	74.000	85.13
	4	80	305	603	75.375	106.14
2003	1	90	308	613	76.625	117.43
	2	72	313	521	77.625	92.75
	3	66	323	636	79.500	83.02
	4	85	329	652	81.500	104.29
2004	1	100	335	664	84.750	92.03
	2	78	343	678	84.750	92.03
	3	72	—	—	—	—
	4	93	—	—	—	—

Table 16.19 Calculation of Seasonal Index

Year	*Quarters* I	II	III	IV
2001	—	—	85.21	90.25
2002	128.12	91.71	85.13	106.14
2003	117.45	92.75	85.13	104.29
2004	120.48	92.03	—	—
Total	366.05	276.49	255.47	300.68
Seasonal average	91.51	69.13	63.87	75.17 = 299.66
Adjusted seasonal index	122.07	92.22	85.20	100.30 ≡ 400

The total of seasonal averages is 299.66. Therefore the corresponding correction factor would be 400/299.68 = 1.334. Each seasonal average is multiplied by the correction factor 1.334 to get the adjusted seasonal indexes shown in Table 16.19.

Example 16.20: Calculate the seasonal indexes by the ratio-to-moving average method from the following data:

Year	*Quarter*	*Actual Values* $(Y = T.C.S.I)$	*4-quarterly Moving Average*	*Year*	*Quarter*	*Given Values* (Y)	*4-quarterly Moving Average*
2000	1	75	—	2002	1	90	76.625
	2	60	—		2	72	77.625
	3	54	63.375		3	66	79.500
	4	59	65.375		4	85	81.500
2001	1	86	67.125	2003	1	100	83.000
	2	65	70.875		2	78	84.750
	3	63	74.000		3	72	—
	4	80	75.375		4	93	—

Solution: Calculations of ratio-to-moving averages are shown in Table 16.20.

Table 16.20 Calculation of Seasonal Indexes

Year	*Quarter*	*Actual Values* ($Y = T.C.S.I$)	*4-quarterly Moving* ($T.C$)	*Ratio to Moving Average (Percentage)* $\frac{Y}{T.C} \times 100$
2000	1	75	—	—
	2	60	—	—
	3	54	63.375	85.21
	4	59	65.375	90.25
2001	1	86	67.125	128.12
	2	65	70.875	91.71
	3	63	74.000	85.14
	4	80	75.375	106.14
2002	1	90	76.625	117.46
	2	72	77.625	92.75
	3	66	79.500	83.02
	4	85	81.500	104.29
2003	1	100	83.000	120.84
	2	78	84.750	92.04
	3	72	—	—
	4	93	—	—

Rearranging the percentages to moving averages, the seasonal indexes are calculated as shown in Table 16.21.

Table 16.21 Seasonal Indexes

Year	*Quarter (Percentages to Moving Averages)*			
	1	*2*	*3*	*4*
2000	—	—	85.21	90.25
2001	128.12	91.71	85.14	106.14
2002	117.46	92.75	83.02	104.30
2003	120.48	92.04	—	—
Total	366.06	276.50	253.37	300.69
Average	122.02	92.17	84.46	100.23 = 398.88
Adjusted seasonal index	$\frac{122.02}{99.72} \times 100$ = 122.36	$\frac{92.17}{99.72} \times 100$ = 92.43	$\frac{84.46}{99.72} \times 100$ = 84.70	$\frac{100.23}{99.72} \times 100$ = 100.51 = 400

Since the total of average indexes is less than 400, the adjustment of the seasonal index has been done by calculating the grand mean value as follows:

$$\bar{\bar{x}} = \frac{122.02 + 92.17 + 84.46 + 100.23}{4} = 99.72$$

The seasonal average values are now converted into adjusted seasonal indexes using $\bar{\bar{x}}$ = 99.72 as shown in Table 16.21.

Advantages and Disadvantages of Ratio-to-Moving Average Method This is the most widely used method for measuring seasonal variations because it eliminates both trend and cyclical variations from the time-series. However, if cyclical variations are not regular, then this method is not capable of eliminating them completely. Seasonal indexes calculated by this method will contain some effect of cyclical variations.

The only disadvantage of this method is that six data values at the beginning and the six data values at the end are not taken into consideration for calculation of seasonal indexes.

16.10.5 Link Relative Method

This method is also known as ***Pearson's method***. The percentages obtained by this method are called ***link relatives*** as these link each month to the preceding one. The steps involved in this method are summarized below:

(i) Convert the monthly (or quarterly) data into link relatives by using the following formula:

$$\text{Link relative for a particular month} = \frac{\text{Data value of current month}}{\text{Data value of preceding month}} \times 100$$

(ii) Calculate the average of link relatives of each month using either median or arithmetic mean.

(iii) Convert the link relatives (L.R.) into chain relatives (C.R.) by using the formula:

$$\text{C.R. for a particular month} = \frac{[\text{L.R. of current month (or quarter)} \times \text{C.R. of preceding month (or quarter)}]}{100}$$

The C.R. for the first month (or quarter) is assumed to be 100.

(iv) Compute the new chain relative for January (first month) on the basis of December (last month) using the formula:

$$\text{New C.R. for January} = \frac{\text{C.R. of January} \times \text{C.R. of December}}{100}$$

The new C.R. is usually not equal to 100 and therefore needs to be multiplied with the monthly correction factor

$$d = \frac{1}{12}(\text{New C.R. for January} - 100)$$

If the figures are given quarterly, then the correction factor would be

$$d = \frac{1}{4}(\text{New C.R. of first quarter} - 100)$$

The corrected C.R. for other months can be calculated by using the formula:

$$\text{Corrected C.R. for } k\text{th month} = \text{Original C.R. of kth month} - (k - 1)\, d$$

where $k = 1, 2, 3, ..., 12$

(v) Find the mean of the corrected chain index. If it is 100, then the corrected chain indexes represent the seasonal variation indexes. Otherwise divide the corrected C.R. of each month (or quarter) by the mean value of corrected C.R. and then multiply by 100 to get the seasonal variation indexes.

Example 16.21: Apply the method of link relatives to the following data and calculate seasonal indexes.

Year	*Quarters*			
	I	*II*	*III*	*IV*
1999	68	62	61	63
2000	65	58	56	61
2001	68	63	63	67
2002	70	59	56	62
2003	60	55	51	58

Solution: Computations of link relatives (L.R.) are shown in Table 16.22 by using the following formula:

$$\text{Link relative of any quarter} = \frac{\text{Data value of current quarter}}{\text{Data value of preceeding quarter}} \times 100$$

Table 16.22 Compution of Link Relatives

Year	Quarters			
	I	II	III	IV
1999	—	91.18	98.39	103.28
2000	103.18	89.23	96.55	108.93
2001	111.48	92.65	100.00	106.35
2002	104.48	84.29	94.91	110.71
2003	96.78	91.67	92.73	113.73
Total of L.R.	415.92	449.02	482.58	543.00
Arithmetic mean of L.R.	103.98	89.80	96.52	108.60
Chain relatives (C.R.)	100	$\frac{89.80 \times 100}{100}$ = 89.80	$\frac{96.52 \times 89.80}{100}$ = 86.67	$\frac{108.60 \times 86.67}{100}$ = 94.12

The new chain relatives for the first quarter on the basis of last quarter is calculated as follows:

$$\text{New C.R} = \frac{\text{L.R. of first quarter} \times \text{C.R. of previous quarter}}{100} = \frac{103.98 \times 94.12}{100} = 97.9$$

Since new C.R. is not equal to 100, therefore we need to apply quarterly correction factor as:

$$d = \frac{1}{4}\ (\text{New C.R. of first quarter} - 100)$$

$$= \frac{1}{4}\ (97.9 - 100) = -0.53$$

Thus the corrected (or adjusted) C.R. for other quarters is shown in Table 16.23. For this we use the formula:

$$\text{Corrected C.R. for kth quarter} = \text{Original C.R. of kth quarter} - (k - 1)\, d$$

where $k = 1, 2, 3, 4$.

Table 16.23 Calculation of Link Relatives

Quarter	I	II	III	IV
Corrected C.R.	100	89.80 – (– 0.53) = 90.33	86.67 – 2(– 0.53) = 87.73	94.13 – 3(– 0.53) = 95.71
Seasonal indexes	$\frac{100}{93.44} \times 100$ = 107.02	$\frac{90.33}{93.44} \times 100$ = 96.67	$\frac{87.73}{93.44} \times 100$ = 93.89	$\frac{95.71}{93.44} \times 100$ = 102.42

$$\text{Mean of corrected C.R.} = \frac{100 + 90.33 + 87.73 + 95.71}{4} = 93.44$$

$$\text{Seasonal variation index} = \frac{\text{Corrected C.R.}}{\text{Mean of corrected C.R.}} \times 100$$

Example 16.22: Apply the method of link relatives to the following data and calculate the seasonal index:

Year	*Quarters*			
	I	*II*	*III*	*IV*
2000	45	54	72	60
2001	48	56	63	56
2002	49	63	70	65
2003	52	65	75	72
2004	60	70	84	86

Solution : Computations of link relatives (L.R.) using the following formula are shown in Table 16.24.

$$\text{L.R. of any quarter} = \frac{\text{Data value of current quarter}}{\text{Data value of preceding quarter}} \times 100$$

Table 16.24 Computation of Link Relatives

Year	*Quarters* I	II	III	IV
2000	—	120	133.33	83.33
2001	80.00	116.67	112.50	88.89
2002	87.50	128.57	111.11	92.86
2003	80.00	125.00	115.38	96.00
2004	85.71	116.67	120.00	78.57
Total of L.R.	333.21	606.91	592.32	439.65
Arithmetic mean of L.R.	83.30	121.38	118.46	87.93
Chain relatives (C.R.)	100	$\frac{121.38 \times 100}{100}$ = 121.38	$\frac{118.46 \times 121.38}{100}$ = 143.78	$\frac{87.93 \times 143.78}{100}$ = 126.42

The new chain relatives for the first quarter on the basis of the preceding quarter is calculated as follows:

$$\text{New C.R.} = \frac{\text{L.R. of first quarter} \times \text{C.R. of previous quarter}}{100}$$

$$= \frac{83.30 \times 126.42}{100} = 105.30$$

Since the new C.R. is more than 100, therefore we need to apply a quarterly correction factor as :

$$d = \frac{1}{4}\ (\text{New C.R. of first quarter} - 100)$$

$$= \frac{1}{4}\ (105.30 - 100) = 1.325$$

Thus the corrected (or adjusted) C.R. for other quarters is shown in Table 16.25. For this we use the formula

Corrected C.R. for kth quarter = Original C.R. of kth quarter – $(k - 1)\,d$

where k = 1, 2, 3, 4.

Table 16.25 Corrected C.R.

Quarters	I	II	III	IV
Corrected C.R.	100	121.38 – 1.32 = 120.06	143.78 – 2(1.32) = 141.14	126.42 – 3(1.32) = 122.46
Seasonal indexes	$\frac{100}{120.92} \times 100$ = 82.70	$\frac{120.06}{120.92} \times 100$ = 99.30	$\frac{141.14}{120.92} \times 100$ = 116.72	$\frac{122.46}{120.92} \times 100$ = 101.27

$$\text{Mean of corrected C.R.} = \frac{100 + 120.06 + 141.14 + 122.46}{4} = 120.92$$

$$\text{Seasonal variation index} = \frac{\text{Corrected C.R.}}{\text{Mean of corrected C.R.}} \times 100$$

Advantages and Disadvantages of Link Relative Method This method is much simpler than the ratio-to-trend or the ratio-to-moving average methods. In this method the L.R. of the first quarter (or month) is not taken into consideration as compared to ratio-to-trend method, where 6 values each at the beginning and at the end periods (month) are lost.

This method eliminates the trend but it is possible only if there is a straight line (linear) trend in the time-series—which is generally not formed in business and economic series.

16.11 MEASUREMENT OF CYCLICAL VARIATIONS—RESIDUAL METHOD

As mentioned earlier that a typical time-series has four components: secular trend (T), seasonal variation (S), cyclical variation (C), and irregular variation (I). In a multiplicative time-series model, these components are written as:

$$y = T.\ C.\ S.\ I$$

The deseasonalization data can be adjusted for trend analysis by dividing these by the corresponding trend and seasonal variation values. Thus we are left with only cyclical (C) and irregular (I) variations in the data set as shown below:

$$\frac{y}{T.S} = \frac{T.C.S.I}{T.S} = C.\ I$$

The moving averages of an appropriate period may be used to eliminate or reduce the effect of irregular variations and thus left behind only the cyclical variations.

The procedure of identifying cyclical variation is known as the *residual method.* Recall that cyclical variations in time-series tend to oscillate above and below the secular trend line for periods longer than one year. The steps of residual method are summarized as follows:

(i) Obtain seasonal indexes and deseasonalized data.

(ii) Obtain trend values and expressed seasonalized data as percentages of the trend values.

(iii) Divide the original data (y) by the corresponding trend values (T) in the time-series to get S. C. I. Further divide S. C. I by S to get C. I.

(iv) Smooth out irregular variations by using moving averages of an appropriate period but of short duration, leaving only the cyclical variation.

16.12 MEASUREMENT OF IRREGULAR VARIATIONS

Since irregular variations are random in nature, no particular procedure can be followed to isolate and identify these variations. However, the residual method can be extended one step further by dividing C. I by the cyclical component (C) to identify the irregular component (I).

Alternately, trend (T), seasonal (S), and cyclical (C) components of the given time-series are estimated and then the residual is taken as the irregular variation. Thus, in the case of multiplicative time-series model, we have

$$\frac{Y}{T.C.S} = \frac{T.C.S.I}{T.C.S} = I$$

where S and C are in fractional form and not in percentages.

Conceptual Questions 16B

15. (a) Under what circumstances can a trend equation be used to forecast a value in a series in the future? Explain.

(b) What are the advantages and disadvantages of trend analysis? When would you use this method of forecasting?

16. What effect does seasonal variability have on a time-series? What is the basis for this variability for an economic time-series?

17. What is measured by a moving average? Why are 4-quarter and 12-month moving averages used to develop a seasonal index?

18. Briefly describe the moving average and least squares methods of measuring trend in time-series.

[*CA, May 1997*]

19. Explain the simple average method of calculating indexes in the context of time-series analysis.

20. Distinguish between ratio-to-trend and ratio-to-moving average as methods of measuring seasonal variations. Which is better and why?

21. Distinguish between trend, seasonal variations, and cyclical variations in a time-series. How can trend be isolated from variations?

22. Describe any two important methods of trend measurement, and examine critically the merits and demerits of these methods.

23. Why do we deseasonalize data? Explain the ratio-to-moving average method to compute the seasonal index.

24. Explain the following:

(a) '. . ., the business analyst who uses moving averages to smoothen data, while in the process of trying to discover business cycles, is likely to come up with some non-existent cycles'.

(b) 'Despite great limitations of statistical forecasting, the forecasting techniques are invaluable to the economist, the businessman, and the Government.'

25. 'A 12-month moving average of time-series data removes trend and cycle'. Do you agree ? Why or why not?

26. Why do we deseasonalize data? Explain the ratio-to-moving average method to compute the seasonal index.

27. Explain the methods of fitting of the quadratic and exponential curves. How would you use the fitted curves for forecasting?

28. 'A key assumption in the classical method of time-series analysis is that each of the component movements in the time-series can be isolated individually from a series'. Do you agree with this statement? Does this assumption create any limitation to such analysis?

Self-Practice Problems 16C

16.19 Apply the method of link relatives to the following data and calculate seasonal indexes.

Quarter	*1999*	*2000*	*2001*	*2002*	*2003*
I	6.0	5.4	6.8	7.2	6.6
II	6.5	7.9	6.5	5.8	7.3
III	7.8	8.4	9.3	7.5	8.0
IV	8.7	7.3	6.4	8.5	7.1

16.20 A company estimates its sales for a particular year to be Rs 24,00,000. The seasonal indexes for sales are as follows:

Month	*Seasonal Index*	*Month*	*Seasonal Index*
January	75	July	102
February	80	August	104
March	98	September	100
April	128	October	102
May	137	November	82
June	119	December	73

Using this information, calculate estimates of monthly sales of the company. (Assume that there is no trend).

[*Osmania Univ., MBA, 1997*]

16.21 Calculate the seasonal index from the following data using the average method:

Year	*Quarter*			
	I	*II*	*III*	*IV*
2000	72	68	80	70
2001	76	70	82	74
2002	74	66	84	80
2003	76	74	84	78
2004	78	74	86	82

[*Kerala Univ., BCom, 1996*]

16.22 Calculate seasonal index numbers from the following data:

Year	*Quarter*			
	I	*II*	*III*	*IV*
1998	108	130	107	93
1999	86	120	110	91
2000	92	118	104	88
2001	78	100	94	78
2002	82	110	98	86
2003	106	118	105	98

16.23 Calculate seasonal index for the following data by using the average method:

Year	*Quarters*			
	I	*II*	*III*	*IV*
2000	72	68	80	70
2001	76	70	82	74
2002	74	66	84	80
2003	76	74	84	78
2004	78	74	86	82

16.24 On the basis of quarterly sales (Rs in lakh) of a certain commodity for the years 2003—2004, the following calculations were made:

Trend : $y = 20 + 0.5\,t$ with origin at first quarter of 2003

where t = time unit (one quarter),

y = quarterly sales (Rs in lakh)

Seasonal variations:

Quarter :	1	2	3	4
Seasonal index :	80	90	120	110

Estimate the quarterly sale for the year 2003 using multiplicative model.

Hints and Answers

16.19

Year	*Quarters*			
	I	*II*	*III*	*IV*
1999	—	108.3	120.0	111.5
2000	62.1	146.3	106.3	89.9
2001	93.2	95.6	143.1	68.8
2002	112.5	80.6	129.3	113.3
2003	77.6	110.6	109.6	88.8
Arithmetic average	$\frac{345.4}{4} = 86.35$	$\frac{541.4}{5} = 108.28$	$\frac{608.3}{5} = 121.66$	$\frac{469.3}{5} = 93.86$
Chain relatives	100	$\frac{100 \times 108.28}{100} = 108.28$	$\frac{121.66 \times 108.28}{100} = 131.73$	$\frac{93.86 \times 131.73}{100} = 123.65$
Corrected chain relatives	100	$108 - 1.675 = 106.325$	$131.73 - 3.35 = 128.38$	$123.64 - 5.025 = 118.615$
Seasonal indexes	$\frac{100 \times 100}{113.4} = 88.18$	$\frac{106.605}{113.4} \times 100 = 94.01$	$\frac{128.38}{113.4} \times 100 = 113.21$	$\frac{118.615}{113.4} \times 100 = 104.60$

16.20 Seasonal indexes are usually expressed as percentages. The total of all the seasonal indexes is 1200.

Seasonal effect = Seasonal index ÷ 100

The yearly sales being Rs 24,00,000, the estimated monthly sales for a specified month:

$$\text{Estimated sales} = \frac{\text{Annual sales}}{12} \times \text{Seasonal effect}$$

$$= \frac{24,00,000}{12} \times \text{Seasonal effect}$$

$$= 2,00,000 \times \text{Seasonal effect}$$

Month (1)	*Seasonal Index* (2)	*Seasonal Effect* (3) = (2) ÷ 100	*Estimated Sales* (4) = (3) × 2,00,000
January	75	0.75	1,50,000
February	80	0.80	1,60,000
March	98	0.98	1,96,000
April	128	1.28	2,56,000
May	137	1.37	2,74,000
June	119	1.19	2,38,000
July	102	1.02	2,04,000
August	104	1.04	2,08,000
September	100	1.00	2,00,000
October	102	1.02	2,04,000
November	82	0.82	1,64,000
December	73	0.73	1,46,000
	1200	12.00	24,00,000

16.21

Year	*Quarterss*			
	I	*II*	*III*	*IV*
2000	72	68	80	70
2001	76	70	82	74
2002	74	66	84	80
2003	76	74	84	78
2004	78	74	86	82
Total	376	352	416	384
Average	75.2	70.4	83.2	76.8
Seasonal index	98.43	92.15	108.9	100.52

$$\text{Grand average} = \frac{75.2 + 70.4 + 83.2 + 76.8}{4}$$

$$= \frac{305.6}{4} = 76.4$$

Seasonal index for quarter

$$k = \frac{\text{Average of quarter } k}{\text{Grand average}} \times 100$$

16.22

Year	*Quarters*			
	I	*II*	*III*	*IV*
1998	108	130	107	93
1999	86	120	110	91
2000	92	118	104	88
2001	78	100	94	78
2002	82	110	98	86
2003	106	118	105	98
Total	552	696	618	534
Average	92	116	103	89
Seasonal Index	$\frac{92}{100}\times100 = 92$	$\frac{116}{100}\times100 = 116$	$\frac{103}{100}\times100 = 103$	$\frac{89}{100}\times100 = 89$

Sales in different quarters:

I: Rs 20,000; II: 20,000×1.16 = Rs 23,200;
III: 20,000 × 1.03 = Rs 20,600;
IV: 20,000 × 0.89 = Rs 17,800

16.23

Year	*Quarters*			
	I	*II*	*III*	*IV*
2000	72	68	80	70
2001	76	70	82	74
2002	74	66	84	80
2003	76	74	84	78
2004	78	74	86	82
Total	376	352	416	384
Average	75.2	70.4	83.2	76.8
Seasonal Index	$\frac{75.2}{76.4}\times 100 = 98.43$	$\frac{70.4}{76.4}\times 100 = 92.15$	$\frac{83.2}{76.4}\times 100 = 108.90$	$\frac{76.8}{76.4}\times 100 = 100.52$

16.24

Quarter of 2003	*Time Unit*	*Trend (T) Values* $y = 20 + 0.5t$	*Seasonal Effect or Seasonal Index* (*S*)	*Estimated Sales (Rs in lakh)* $T \cdot S$
1	4	20 + 0.5 × 4 = 22.0	0.80	17.60
2	5	20 + 0.5 × 5 = 22.5	0.90	20.25
3	6	20 + 0.5 × 6 = 23.0	1.20	27.60
4	7	20 + 0.5 × 7 = 23.5	1.10	25.85

Formulae Used

1. Secular trend line
   - Linear trend model

     $$y = a + bx$$

     where $a = \bar{y} - b\bar{x}$; $b = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n(\bar{x})^2}$
   - Exponential trend model

     $$y = ab^x;$$

     $$\log a = \frac{1}{n}\,\Sigma \log y;\quad \log b = \frac{\Sigma x \log y}{\Sigma x^2}$$
   - Parabolic trend model

     $$y = a + bx + cx^2$$

     where $a = \frac{\Sigma y - c\Sigma x^2}{n}$; $b = \frac{\Sigma xy}{\Sigma x^2}$

     $$c = \frac{n\Sigma x^2 y - \Sigma x^2 \Sigma y}{n\Sigma x^4 - (\Sigma x^2)^2}$$

2. Moving average

   $$MA_{t+1} = \frac{\Sigma\{D_t + D_{t-1} + \ldots + D_{t-n+1}\}}{n}$$

   where t = current time period
   D = actual data value
   n = length of time period

3. Simple exponential smoothing

$$F_t = F_{t-1} + \alpha (D_{t-1} - F_{t-1})$$

where F_t = current period forecast
F_{t-1} = previous period forecast
α = a weight ($0 \le \alpha \le 1$)
D_{t-1} = previous period actual demand

4. Adjusted exponential smoothing

$$(F_t)_{adj} = F_t + \frac{1-\beta}{\beta} T_t$$

where β = smoothing constant for trend
T_t = exponential smoothed trend factor

Chapter Concepts Quiz

True or False

1. Exponential smoothing is an example of a causal model. (T/F)
2. Secular trends represent the long-term direction of a time-series. (T/F)
3. The repetitive movement around a trend line in a one-year period is best described by seasonal variation. (T/F)
4. The present of trend should not be used for predicting future cyclical variations. (T/F)
5. A time-series model incorporates the various factors that might influence the quantity being forecast. (T/F)
6. In exponential smoothing, when the smoothing constant is high, more weight is placed on the more recent data. (T/F)
7. Seasonal variation is a repetitive and predictable variation around the trend line within a year. (T/F)
8. In a trend-adjusted exponential smoothing model, a high value for the trend smoothing constant β implies that we wish to make the model less responsive to recent changes in trend. (T/F)
9. A time-series should be deseasonalized after the trend or cyclical components of the time-series have been identified. (T/F)
10. The weakness of causal forecasting methods is that we must first forecast the value of the independent variable. (T/F)
11. No single forecast methodology is appropriate under all conditions. (T/F)
12. Regression analysis can only be used to develop a forecast based upon a single independent variable. (T/F)
13. Exponential smoothing is a weighted moving average model where all previous values are weighted with a set of weights that decline exponentially. (T/F)
14. Periods of moving averages are determined by the periodicity of the time-series. (T/F)
15. No trend values are lost when determined by the method of moving averages. (T/F)

Multiple Choice

16. Forecasting time horizons include
    (a) long range (b) medium range
    (b) short range (d) all of these
17. A forecast that projects company's sales is the
    (a) economic forecast (b) technological forecast
    (c) demand forecast (d) none of these
18. Quantitative methods of forecasting include
    (a) salesforce composite (b) consumer market survey
    (c) smoothing approach (d) all of these
19. Decomposing a time-series refers to breaking down past data into the components of
    (a) constants and variations
    (b) trends, cycles, and random variations
    (c) tactical and operational variations
    (d) long-term, short-term, and medium-term variations
20. Consider a time-series of data for the quarters of 1995 and 1996. The third quarter of 1996 would be coded as:
    (a) 2 (b) 3 (c) 5 (d) 6
21. If a time-series has an even number of years and we use coding, then each coded interval is equal to
    (a) one month (b) 6 months
    (c) one year (d) two years
22. The cyclical variation in the time-series is eliminated by
    (a) second-degree analysis
    (b) spearman analysis
    (c) relative cyclical residual
    (d) none of these
23. Suppose a time-series is fitted with parabolic trend model $\hat{y} = a + bx + cx^2$. What do the x's represent in this model?
    (a) coded values of the time variables
    (b) variable to be determined
    (c) estimates of the dependent variable
    (d) none of these
24. A component of time-series used for short-term forecast is
    (a) trend (b) seasonal
    (c) cyclical (d) irregular
25. In an additive time-series model, the component measurements are
    (a) positive (b) negative
    (c) absolute (d) none of these

26. After detrending, the time-series multiplicative model is represented as:
(a) Y = T S C I (b) Y = S C I
(c) Y = T S I (d) none of these

27. In a time-series multiplicative model its components S, C, and I have
(a) positive values (b) absolute values
(d) indexing values (d) none of these

28. With regard to a regression-based forecast, the standard error of estimate gives a measure of
(a) overall accuracy of the forecast
(b) time period for which the forecast is valid
(b) maximum error of the forecast
(d) all of these

29. Seasonal indexes are calculated by using
(a) freehand curve method(b) moving average method
(c) link relative method (d) none of these

30. Suppose a time-series is described by the equation $\hat{y} = 10 + 2x + 7x^2$ based on data for the years 1994–2000. What is the forecast value of $\hat{y}$ for the year 2001?
(a) 130 (b) 195 (c) 245 (d) 800

Concepts Quiz Answers

1. F	2. T	3. F	4. T	5. F	6. T	7. T	8. F	9. F
10. T	11. T	12. F	13. T	14. T	15. F	16. (d)	17. (c)	18. (c)
19. (b)	20. (c)	21. (b)	22. (d)	23. (a)	24. (b)	25. (c)	26. (b)	27. (d)
28. (a)	29. (c)	30. (a)						

Review Self-Practice Problems

16.25 A sugar mill is committed to accepting beets from local producers and has experienced the following supply pattern (in thousands of tons/year and rounded).

Year	*Tonnes*	*Year*	*Tonnes*
1990	100	1995	400
1991	100	1996	400
1992	200	1997	600
1993	600	1998	800
1994	500	1999	800

The operations manager would like to project a trend to determine what facility additions will be required by 2004

(a) Sketch a freehand curve and extend it to 2004. What would be your 2004 forecast based upon the curve?

(b) Compute a three-year moving average and plot it as a dotted line on your graph.

16.26 Use the data of Problem 16.25 and the normal equations to develop a least squares line of best fit. Omit the year 1990.

(a) State the equation when the origin is 1995.

(b) Use your equation to estimate the trend value for 2004.

16.27 A forecasting equation is of the form:

$$\hat{y}_c = 720 + 144x$$

$$[2003 = 0, \quad x \text{ unit} = 1 \text{ year}, y = \text{annual sales}]$$

(a) Forecast the annual sales rate for 2003 and also for one year later.

(b) Change the time (x) scale to months and forecast the annual sales rate at July 1, 2003, and also at one year later.

(c) Change the sales (y) scale to monthly and forecast the monthly sales rate at July 1, 2003, and also at one year later.

16.28 Data collected on the monthly demand for an item were as shown below:

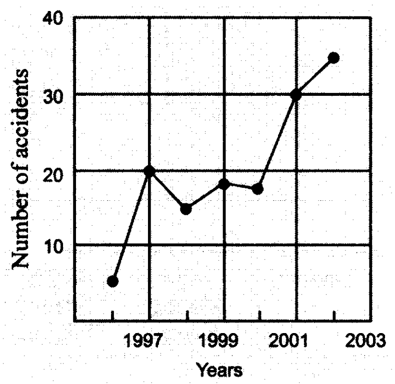
January	100
February	90
March	80
April	150
May	240
June	320
July	300
August	280
September	220

(a) What conclusion can you draw with respect to the length of moving average versus smoothing effect?

(b) Assume that the 12-month moving average centred on July was 231. What is the value of the ratio-to-moving average that would be used in computing a seasonal index?

16.29 The data shown in the table below gives the number of lost-time accidents over the past seven years in a cement factory:

Year	*Number Employees (in '000)*	*Number Accidents*
1996	15	5
1997	12	20
1998	20	15
1999	26	18
2000	35	17
2001	30	30
2002	37	35



(a) Use the normal equations to develop a linear time-series equation forecasting the number of accidents.

(b) Use your equation to forecast the number of accidents in 2005.

16.30 Consider the following time-series data:

Week :	1	2	3	4	5	6
Value :	8	13	15	17	16	9

(a) Develop a 3-week moving average for this time-series. What is the forecast for week 7?

(b) Use $\alpha = 0.2$ to compute the exponential smoothing values for the time-series. What is the forecast for week 7?

16.31 Admission application forms data (1000's) received by a management institute over the past 6 years are shown below:

Year :	1	2	3	4	5	6
Application forms :	20.5	20.2	19.5	19.0	19.1	18.8

Develop the equation for the linear trend component of this time-series. Comment on what is happening to admission forms for this institution.

16.32 Consider the following time-series data:

Quarter	*Year*		
	1	*2*	*3*
1	4	6	7
2	2	3	6
3	3	5	6
4	5	7	8

(a) Show the 4-quarter moving average values for this time-series.

(b) Compute seasonal indexes for the 4 quarters.

16.33 Below are given the figures of production (in million tonnes) of a cement factory:

Year :	1990	1992	1993	1994	1995	1996	1999
Production :	77	88	94	85	91	98	90

(a) Fit a straight line trend by the 'least squares method' and tabulate the trend values.

(b) Eliminate the trend. What components of the time series are thus left over?

(c) What is the monthly increase in the production of cement? [*Sukhadia Univ., MBA, 1999*]

16.34 The sale of commodity in tonnes varied from January 2000 to December, 2000 in the following manner:

280	300	280	280	270	240
230	230	220	200	210	200

Fit a trend line by the method of semi-averages.

16.35 Fit a parabolic curve of the second degree to the data given below and estimate the value for 2002 and comment on it.

Year :	1996	1997	1998	1999	2000
Sales (Rs in '000) :	10	12	13	10	8

16.36 Given below are the figures of production of a sugar (in 1000 quintals) factory:

Year :	1991	1992	1993	1994	1995	1996	1997
Production :	40	45	46	42	47	49	46

Fit a straight line trend by the method of least squares and estimate the value for 2001.

[*MBA, MD Univ., 1998*]

16.37 The following table gives the profits (Rs in thousand) of a concern for 5 years ending 1996.

Year :	1996	1997	1998	1999	2000
Profits :	1.6	4.5	13.8	40.2	125.0

Fit an equation of the type $y = ab^x$.

Hints and Answers

16.25 (a) Forecasts is around 1200 (thousand) tonnes

(b) Averages are: 133, 300, 433, 500, 433, 466, 600 and 733.

16.26 (a) $\hat{y} = 489 + 75x$ [1995 = 0, x = years, y = tonnes in thousand]

(b) 11,64,000 tonnes

16.27 (a) 720 units when $x = 0$, 864 units when $x = 1$.

(b) $\hat{y} = 720 + 12x$ [July 1, 2003 = 0; x unit = 1 month; y = annual sales rates in units]

720 units per year; 864 units per year.

(c) $\hat{y} = 60 + x$ [July 1, 2003 = 0, x unit = 1 month; y = monthly sales rates in units]

60 units per month; 72 units per month.

16.28 (a) Longer average yield more smoothing; (b) 1.3

16.29 (a) $\hat{y} = 20 + 4x$ [1999 = 0, x = years; y = number of accidents]; (b) 44

16.30 (a)

Week (1)	*Values (2)*	*Forecast (3)*	*Forecast Error (4) = (2) − (3)*	*Squared Forecast Error*
1	8	—	—	—
2	13	—	—	—
3	15	—	—	—
4	17	12	5	25
5	16	15	1	1
6	9	16	−7	49

Forecast for week 7 is: (17 + 16 + 9)/3 = 14.

(b)

Week (t)	*Values* y_t	*Forecast* F_t	*Forecast Error* $y_t - F_t$	*Squared Error* $(y_t - F_t)^2$
1	8	—	—	—
2	13	8.00	5.00	25.00
3	15	9.00	6.00	36.00
4	17	10.20	6.80	46.24
5	16	11.56	4.44	19.71
6	9	12.45	−3.45	11.90
				138.85

Forecast for week 7 is: 0.2 (9)+(1 − 0.2) (12.45) = 11.76.

16.31 $\Sigma x = 21, \Sigma x^2 = 91, \Sigma y = 117.1, \Sigma xy = 403.7$

$$b = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} = \frac{6(403.7) - 21 \times 117.2}{6 \times 91 - (21)^2}$$

$= -0.3714$

$a = \bar{y} - b\bar{x} = 19.5167 - (-0.3514)(3.5) = 20.7466$

$\hat{y} = 20.7466 - 0.3514x$.

Enrolment appears to be decreasing by about 351 students per year.

16.32 (a)

Year	*Quarter y*	*Value*	*4-quarter Moving Average*	*Centred Moving Average*
1	1	4		
	2	2	3.50	
	3	3	4.00	3.750
	4	5	4.25	4.125
2	1	6	4.75	4.500
	2	3	5.25	5.000
	3	5	5.50	5.375
	4	7	6.25	5.875
3	1	7	6.50	6.375
	2	6	6.75	6.625
	3	6	—	
	4	8	—	

(b)

Year	*Quarter y*	*Value Moving Average*	*Centered Irregular Component*	*Seasonal*
1	1	4	—	
	2	2	—	
	3	3	3.750	0.8000
	4	5	4.125	1.2121
2	1	6	4.500	1.3333
	2	3	5.000	0.6000
	3	5	5.375	0.9302
	4	7	5.875	1.1915
3	1	7	6.375	1.0000
	2	6	6.625	0.9057
	3	6	—	—
	4	8	—	—

Quarter	*Seasonal-Irregular Component Values*	*Seasonal Index*
1	1.333, 1.0980	1.2157
2	0.6000, 0.9057	0.7529
3	0.8000, 0.9302	0.8651
4	1.2121, 1.1915	1.2018
		4.0355

Ajusted for seasonal index $= \frac{4}{4.0355} = 0.9912$.

16.33 (a)

Year	Time Period	Production (in m. tonnes)	Deviation From 1994			Trend Values
		y	x	xy	x	$\hat{y}$
1990	– 4	77	– 4	– 308	16	83.299
1992	– 2	88	– 2	– 176	4	86.051
1993	– 1	94	– 1	– 94	1	87.427
1994	0	85	0	0	0	88.803
1995	1	91	1	91	1	90.179
1996	2	98	2	196	4	91.555
1999	5	90	5	450	25	95.683
		623	1	159	51	

Solving the normal equations

$\Sigma y = na + b\Sigma x$ $\qquad 623 = 7a + b$

$\Sigma xy = a\Sigma x + b\Sigma x^2$ $\qquad 159 = a + 5b$

we get $a = 88.803$ and $b = 1.376x$. Thus

$$\hat{y} = a + bx = 88.803 + 1.376x$$

Substituting $x = -4, -2, -1, 0, 1, 2, 5$ to get trend values as shown above in the table.

(b) After eliminating the trend, we are left with S, C, and I components of time-series.

(c) Monthly increase in the production of cement in given by $b/12 = 1.376/12 = 0.115$.

16.34

Month (in tonnes)	Sales	
January	280	Total = 1650 of first six months;
February	300	
March	280	
April	280	Average $= \dfrac{1650}{6} = 275$
May	270	
June	240	
July	230	Total = 1290 of last six months;
August	230	
September	220	
October	200	Average $= \dfrac{1290}{6} = 215$

Plot 275 and 215 in the middle of March-April 2000 and that of September-October 2000. By joining these two points we get a trend line which describes the given data.

16.35

Year	Sales	Period				
	y	x	xy	x^2	x^2y	x^4
1996	10	– 2	– 20	4	40	16
1997	12	– 1	– 12	1	12	1
1998	13	0	0	0	0	0
1999	10	1	10	1	10	1
2000	8	2	16	4	32	16
	53	0	– 6	10	94	34

Parabolic trend line : $y = a + bx + bx^2$

$$a = \frac{\Sigma y - c\Sigma x^2}{n} = \frac{53 - 0.857 \times 10}{5} = 8.886$$

$$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{-6}{10} = -0.6\,;$$

$$c = \frac{n\Sigma x^2 y - \Sigma x^2\, \Sigma y}{n\Sigma x^4 - (\Sigma x^2)^2} = \frac{5(94) - 10(53)}{5(34) - (10)^2} = -0.857$$

$\therefore\ y = 8.886 - 0.6x - 0.857x^2$

For 2002, $x = 4$; $y = 8.886 - 0.6\,(4) - 0.857\,(4)^2$

$= -7.226$

16.36

Year	Production ('000 qtls)	Deviations from 1994		
	y	x	xy	x^2
1991	40	– 3	– 120	9
1992	45	– 2	– 90	4
1993	46	– 1	– 46	1
1994	42	0	0	0
1995	47	1	47	1
1996	49	2	98	4
1997	46	3	138	9
	315	0	27	28

$\hat{y} = a + bx$; $a = \Sigma y/n = 315/7 = 45$;

$$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{27}{28} = 0.964$$

$\hat{y} = 45 + 0.964x$

$y_{2001} = 45 + 0.964\,(7) = 45 + 6.748 = 51.748$

16.37

Year	Profits y	x	Log y	x^2	x . Log y
1996	1.6	– 2	0.2041	4	– 0.4082
1997	4.5	– 1	0.6532	1	– 0.6532
1998	13.8	0	1.1399	0	0
1999	40.2	1	1.6042	1	1.6042
2000	125.0	2	2.0969	4	4.1938
	185.1	0	5.6983	10	4.7366

Trend line: $y = ab^x$ or $\log y = \log a + x \log b$

where $\log a = \dfrac{\Sigma \log y}{n} = \dfrac{5.6983}{5} = 1.1397;$

$$\log b = \frac{\Sigma x \log y}{\Sigma x^2} = \frac{4.7366}{10} = 0.474$$

Thus $\log y = 1.1397 + 0.474\,x$.

And in such indexes ..., there is seen the baby figure of the gaint mass of things to come.
—William Shakespeare

Chapter 17

Index Numbers

LEARNING OBJECTIVES

After studying this chapter, you should be able to

- explain the purpose of index numbers.
- compute indexes to measure price changes and quantity changes over time.
- revise the base period of a series of index numbers
- explain and derive link relatives
- discuss the limitations of index number construction

17.1 INTRODUCTION

We know that most values change and therefore may want to know-how much change has taken place over a period of time. For example, we may want to know-how much the prices of different items essential to a household have increased or decreased so that necessary adjustments can be made in the monthly budget. An organization may be concerned with the way in which prices paid for raw materials, annual income and profit, commodity prices, share prices, production volume, advertising budget, wage bills, and so on, have changed over a period of time. However, while prices of a few items may have increased, others may have decreased over a given period of time. Consequently in all such situations, an average measure needs to be defined to compare such differences from one time period to another. *Index numbers* are yardsticks for describing such difference.

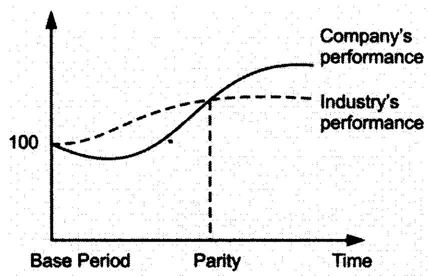
An *index number* can be defined as a relative measure describing the average changes in any quantity over time. In other words, an index number measures the changing value of prices, quantities, or values over a period of time in relation to its value at some fixed point in time, called the *base period*. This resulting ratio of the current value to a base value is multiplied by 100 to express the index as a percentage. Since an index number is constructed as a ratio of a measure taken during one time period to that same measure taken during another time period (called base period), it has no unit and is always expressed as a percentage term as follows:

$$\text{Index number} = \frac{\text{Current period value}}{\text{Base period value}} \times 100$$

Indexes may be based at any convenient period, which is occasionally adjusted, and these are published at any convenient frequency. Examples of some indexes are:

Daily	Stock market prices
Monthly	Unemployment figures
Yearly	Gross National Product (GNP)

Figure 17.1
Graph of Two Indexes



Index numbers were originally developed by economists for monitoring and comparing different groups of goods. For decision-making in business, it is sometimes essential to understand and manipulate different published index series and to construct one's own index series. This index series can be compared with a national one and/or with competitor's. For example, a cement company could construct an index of its own sales and production volumes and compare it to the index of the cement industry. A graph of two indexes will provide, at a glace, a view of a company's performance within the industry, as shown in Fig. 17.1.

17.2 INDEX NUMBER DEFINED

Definition of index numbers can be classified into the following three broad categories:

1. **A measure of change**
   - It is a numerical value characterizing the change in complex economic phenomena over a period of time or space.` —Maslow
   - An index number is a quantity which, by reference to a base period, shows by its variations the changes in the magnitude over a period of time. In general, index numbers are used to measure changes over time in magnitudes which are not capable of direct measurement. —John I. Raffin
   - An index number is a statistical measure designed to show changes in variables or a group of related variables with respect to time, geographic location or other characteristics. —Speigel
   - Index number is a single ratio (usually in percentages) which measures the combined (i.e., averaged change of several variables between two different times, places or situations. —A. M. Tuttle
2. **A device to measure change**
   - Index numbers are devices measuring differences in the magnitude of a group of related variables. —Corxton and Cowden
   - An index number is a device which shows by its variation the changes in a magnitude which is not capable of accurate measurement in itself or of direct valuation in practice. —Wheldom
3. **A series representing the process of change**
   - Index numbers are series of numbers by which changes in the magnitude of a phenomenon are measured from time to time or place to place. —Horace Secris
   - A series of index numbers reflects in its trend and fluctuations the movements of some quantity of which it is related. —B. L. Bowley
   - An index number is a statistical measure of fluctuations in a variable arranged in the form of a series, and using a base period for making comparisons. —L. J. Kaplan

17.3 TYPES OF INDEX NUMBERS

Index numbers are broadly classified into three categories: (i) price indexes, (ii) quantity indexes, and (iii) value indexes. A brief description of each of these is as follows:

Price Indexes These indexes are of two categories:

- Single price index
- Composite prices index

The single price index measures the precentage change in the current price per unit of a product to its base period price. To facilitate comparisons with other years, the actual per unit price is converted into a *price relative*, which expresses the unit price in each period as a percentage of unit price in a base period. Price relatives are very helpful to understand and interpret changing economic and business conditions over time. Table 17.1 illustrates the calculations of price relatives,

Table 17.1: Calculation of Price Index (Base year = 1996)

Year	*Total Wage Bill (Rs millions)*	*Ratio*	*Price Index or Percentage Relative*
(1)	*(2)*	*(3) = (2)/11.76*	*(4) = (3) x 100*
2000	11.76	11.76/11.76 = 1.0	100.0
2001	12.23	12.23/11.76 = 1.039	103.9
2002	12.84	12.84/11.76 = 1.091	109.1
2003	13.35	13.35/11.76 = 1.135	113.5
2004	13.82	13.82/11.76 = 1.175	117.5

From Table 17.1, it is observed that the price relative of 113.5 in 2003 shows a increase of 13.5% in wage bill compared to the base year 2000.

A composite price index measures the average price change for a basket of related items from a base period to the current period. For example, the *wholesale price index* reflects the general price level for a group of items (or a basket of items) taken as a whole.

The *retail price index* reflects the general changes in the retail prices of various items including food, housing, clothing, and so on. In India, the Bureau of Labour statistics, publishes retail price index. The consumer price index, a special type of retail price index, is the primary measure of the cost of living in a country. The consumer price index is a weighted average price index with fixed weights. The weightage applied to each item in the basket of items is derived from the urban and rural families.

Quantity Index A quantity index measures the relative changes in quantity levels of a group (or basket) of items consumed or produced, such as agricultural and industrial production, imports and exports, between two time periods. The method of constructing quantity indexes is the same as that of price index except that the quantities are vary from period to period.

Quantity index: An index that is constructed to measure changes in quantities over time.

The two most common quantity indexes are the weighted relative of aggregates and the weighted average of quantity relative index.

Value Index A value index measures the relative changes in total monetary worth of an item, such as inventories, sales, or foreign trade, between the current and base periods. The value of an item is determined by multiplying its unit price by the quantity under consideration. The value index can also be used to measure differences in a given variable in different locations. For example, the comparative cost of living shows that in terms of cost of goods and services, it is cheaper to live in a small city than in metro cities.

Special Purpose Indexes A few index numbers such as industrial production, agricultural production, productivity, etc. can also be constructed separately depending on the nature and degree of relationship between groups and items.

- Index number, almost alone in the domain of social sciences, may truly be called an exact science, if it be permissible to designate as science the theoretical foundations of a useful art. —Irving Fisher.

17.4 CHARACTERISTICS AND USES OF INDEX NUMBERS

Based on the definitions and types of index numbers discussed earlier in this chapter, the following characteristics and uses of index number emerge.

17.4.1 Characteristics of Index Numbers

1. **Index numbers are specialized averages:** According to R. L. Corner, '*An index number represents a special case of an average, generally weighted average, compiled from a sample of items judged to be representative of the whole*'.

   'Average' is a single figure representing the characteristic of a data set. This figure can be used as a basis for comparing two or more data sets provided the unit of measurement of observations in all sets is the same. However, index numbers which are considered as a special case of average can be used for comparison of two or more data sets expressed in different units of measurement.

   The consumer price index, for example, which represents a price comparison for a group of items—food, clothing, fuel, house rent, and so on, are expressed in different units. An average of prices of all these items expressed in different units is obtained by using the technique of price index number calculation.

Consumer price index: A price index that uses the price changes in a market basket of consumer goods and services to measure the changes in consumer prices over time.

2. **Index numbers measure the change in the level of phenomena in percentages:** Since index numbers are considered as a special case of an average, these are used to represent, in one single figure, the increase or decrease (expressed in terms of percentage) in the value of a variable. For example, a quantity index number of 110 for cars sold in a given year when compared with that of a base year would mean that cars sales in the given year were 10 per cent higher than in the base year (value of index number in base period is always equal to 100). Similarly, a quantity index number of 90 in a given year would indicate that the number of cars sold in the given were 10 per cent less than in the base year.

3. **Index numbers measure changes in a variety of phenomena which cannot be measured directly:** According to Bowley, '*Index numbers are used to measure the changes in some quantity which we cannot observe directly. . .*'

   It is not possible, for example, to directly measure the changes in the import-export activities of a country. However, it is possible to study relative changes in import and export activities by studying the variations in factors such as raw materials available, technology, competitors, quality, and other parameters which affect import and export, and are capable of direct measurement. Similarly, cost of living cannot be measured in quantitative terms directly, we can only study relative changes in it by studying the variations in certain other factors connected to it.

4. **Index numbers measure the effect of changes in relation to time or place:** Index numbers are used to compare changes which take place over periods of time, between locations, and in categories. For example, cost of living may be different at two different places at the same or cost of living in one city can be compared across two periods of time.

17.4.2 Uses of Index Numbers

According to G. Simpson and F. Kafka '*Index numbers are today one of the most widely used statistical tools. They are used to feel the pulse of the economy and they have come to be used as indicators of inflationary or deflationary tendencies*'. Other important uses of index number can be summarized as follows:

1. **Index numbers act as economic barometers:** A barometer is an instrument that is used to measure atmospheric pressure. Index numbers are used to feel the pressure of the economic and business behaviour, as well as to measure ups and downs in the general economic condition of a country. For example, the composite index number of indexes of prices, industrial output, foreign exchange reserves, and bank deposits, could act as an economic barometer.

2. **Index numbers help in policy formulation:** Many aspects of economic activity are related to price movements. The price indexes can be used as indicators of change in various segments of the economy. For example, by examining the price indexes of different segments of a firm's operations, the management can assess the impact of price changes and accordingly take some remedial and/or preventive actions.

   In the same way, by examining the population index, the government can assess the need to formulate a policy for health, education, and other utilities.

3. **Index numbers reveal trends and tendencies:** An index number is defined as a relative measure describing the average change in the level of a phenomenon between the current period and a base period. This property of the index number can be used to reflect typical patterns of change in the level of a phenomenon. For example, by examining the index number of industrial production, agricultural production, imports, exports, and wholesale and retail prices for the last 8–10 years, we can draw the trend of the phenomenon under study and also draw conclusions as to how much change has taken place due to the various factors.

4. **Index numbers help to measure purchasing power:** In general, the purchasing power is not associated with a particular individual; rather it is related to an entire class or group. Furthermore, it is not associated with the cost of a single item, because individuals purchase many different items in order to live. Consequently, earnings of a group of people or class must be adjusted with a price index that provides an overall view of the purchasing power for the group.

   For example, suppose a person earns Rs 1000 per month in 1990. If an item costs Rs 100 in that year, the person could purchase $1000 \div 100 = 10$ units of the item with one month's earnings, But if in year 2000, the same person earns Rs 2000 per month but the item cost is Rs 250, then he could purchase $2000 \div 250 = 8$ units of the item. Hence, the effect of monthly earning relative to the particular item is less in year 2000 than in 1990 as a lesser number of units of the items can be purchased with current earnings. By dividing the item price in both the years, we can eliminate the effect of price and determine the real purchasing power for that item. For instance, in 1990, the purchasing power was $10 \div 1000 = 0.10$ or 10 paise which it was Rs 0.125 or 12.5 paise in 2000.

5. **Index numbers help in deflating various values:** When real rupee value is computed, the base period is earlier than the given years for which this value is being determined Thus the adjustment of current rupee value to real terms is referred to as *deflating a value series* because prices typically increase over time.

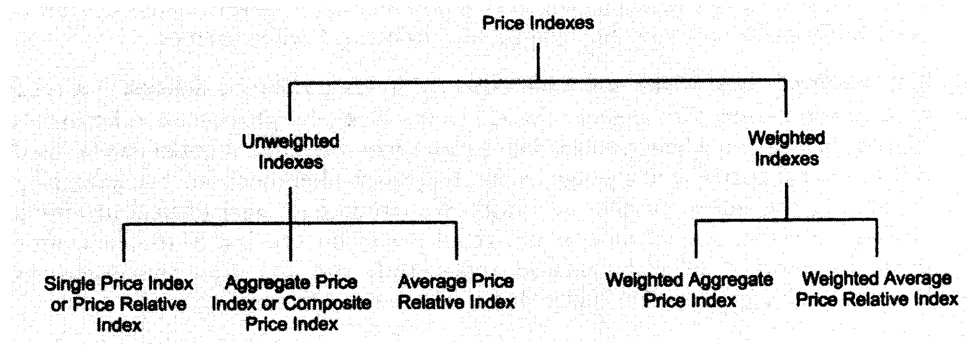
   The price index number is helpful in deflating the national income to remove the effect of inflation over a long term, so that we may understand whether there is any change in the real income of the people or not. The retail price index is often used to compute real changes in earnings and expenditure as it compares the purchasing power of money at different points in time. It is generally accepted as a standard measure of inflation even though calculated from a restricted basket of goods.

Conceptual Questions 17A

1. Explain the significance of index numbers.
2. Explain the differences among the three principal types of indexes: price, quantity, and value.
3. How are index numbers constructed? What is their purpose?
4. What is an index number? Describe briefly its applications in business and industry.
5. What does an index number measure? Explain the nature and uses of index numbers.
6. Index numbers are economic barometers. Explain this statement and mention the limitations of index numbers (if any).
7. What are the basic characteristic of an index number?
8. Since value of the base year is always 100, it does not make any difference which period is selected as the base on which to construct an index. Comment.
9. What are the main uses of an index number?
10. What is meant by the term deflating a value series?

17.5 METHODS FOR CONSTRUCTION OF PRICE INDEXES

Various types of price indexes and their methods of construction can be classified into broad categories as shown in the chart below:



17.6 UNWEIGHTED PRICE INDEXES

The unweighted price indexes are further classified into three groups as shown above in the chart. The method of calculating each of these is discussed below:

17.6.1 Single Price Index

A *single unweighted price index number measures the percentage change in price for a single item or a basket of items between any two time periods*. Unweighted implies that all the values considered in calculating the index are of equal importance.

An unweighted single price index is calculated by dividing the price of an item in the given period by the price of the same item in the base period. To facilitate comparison with other years, the actual price of the item can be converted into a *price relative*, which expresses the unit price in each year (period) as a percentage of the unit price in a base year.

The general formula for calculating the single price index or price relative index is

$$\text{Single price index in period } n = \frac{p_n}{p_0} \times 100$$

where p_n = price per unit of an item in the nth year

p_0 = price per unit of an item in the base year

Example 17.1: The retail price of a typical commodity over a period of four years is given below:

Year	:	2000	2001	2002	2003
Price (Rs)	:	24.60	25.35	26.00	26.50

(a) Find the price index based on 2000 prices

(b) Find the percentage change in price between consecutive years (base year = 2000)

(c) Find the percentage increase between consecutive years

Solution: (a) For the prices of the commodity with base year 2000, the price relatives for one unit of the commodity in the years 2000 to 2003 are given in Table 17.2.

Table 17.2: Price Relatives

Year	*Price (Rs)*	*Price Relatives*	*Percentage Change*
2000	24.60	100	—
2001	25.35	$\frac{25.35}{24.60}\times 100 = 103.04$	3.04
2002	26.00	$\frac{26}{24.60}\times 100 = 105.69$	2.65
2003	26.50	$\frac{26.50}{24.60}\times 100 = 107.72$	2.03

(c) The percentage change in price relative is divided by the index it has come from and multiplied by 100 for finding percentage increase.

For year 2001: $\frac{103.04 - 100}{100} \times 100 = 3.04$ per cent

For year 2002: $\frac{105.69 - 103.04}{103.04} \times 100 = 2.57$ per cent

For year 2003: $\frac{107.72 - 105.69}{105.69} \times 100 = 1.92$ per cent

17.6.2 Aggregate Price Index

An **aggregate index price** or *composite price index measures the average price change for a basket of related items from the base period to the current period.* For example, to measure the change in the cost of living over a period of time, we need the index that measures the change based on the price changes for a variety of commodities including food, housing, clothing, transportation, health care, and so on. Since the number of commodities is large, therefore a sample of commodities should be selected for calculating the aggregate price index.

Aggregate price index: A composite price index based on the prices of a group of commodities or items.

Irrespective of the units of measurement in which prices of several commodities are quoted, the steps of the method to calculate an aggregate price index are summarized as follows:

(i) Add the unit prices of a group of commodities in the year of interest.
(ii) Add the unit prices of a group of commodities in the base year.
(iii) Divide the sum obtained in step (i) by the sum obtained in step (ii), and multiply the quotient by 100.

From the sample of commodities or items included in the calculation of index, we cannot expect a true reflection of price changes for all commodities. This calculation provides us with only a rough estimate of price change.

A formula of calculating an unweighted aggregate price index is defined as:

$$\text{Aggregate price index } P_{01} = \frac{\Sigma p_1}{\Sigma p_0}\times 100 \qquad (17\text{-}2)$$

where p_1 = unit price of a commodity in the current period of interest
p_0 = unit price for a commodity in the base period

Example 17.2: The following are two sets of retail prices of a typical family's shopping basket. The data pertain to retail prices during 2001 and 2002.

Commodity	*Unit Price (Rs)*	
	2001	*2002*
Milk (1 litre)	18	20
Eggs (1 dozen)	15	18
Butter (1 kg)	120	150
Bread (500 gm)	9	11

Calculate the simple aggregate price index for 2002 using 2000 as the base year.

Solution: Calculations for aggregate price index are shown in Table 17.3.

Table 17.3: Calculation of Aggregate Price Index

Commodity	*Unit Price (Rs)*	
	2000 (p_0)	*2002 (p_1)*
Milk (1 litre)	18	20
Egg (1 dozen)	15	18
Butter (1 kg)	120	150
Bread (500 gm)	9	11
Total	162	199

The unweighted aggregate price index for expenses on a few food items in 2002 is given by

$$P_{01} = \frac{\Sigma p_1}{\Sigma p_0} \times 100 = \frac{199}{162} \times 100 = 122.83$$

The value $P_{01} = 122.83$ implies that the price of food items included in the price index has increased by 22.83% over the period 2000 to 2002.

Unweighted aggregate price index: A composite price index in which the price of commodities or items are weighted in accordance of their relative importance.

Limitations of an Unweighted Aggregate Price Index

1. The unweighted aggregate approach of calculating a composite price index is heavily influenced by the items with large per unit price. Consequently items with relatively low unit price are dominated by the high unit price items.
2. Equal weights are assigned to every commodity included in the index irrespective of the relative importance of the commodity in terms of the amount purchased by a typical consumer. In other words, it did not attach more weight or importance to the price change of a high-use commodity than it did to a low-use commodity. For example, a family may purchase 30 packets of 500 gm bread in a month while it is unusual to buy 30 kg butter every month. A substantial price change for slow-moving items like butter, ghee can distort an index.

   Due to these limitations, the unweighted index is not widely used in statistical analyses. These limitations suggest the use of weighted index. There are two methods to calculate weighted index, and these will be discussed later in the chapter.

17.6.3 Average Price Relative Index

This index is an improvement over the aggregate price index because it is not affected by the unit in which prices are quoted. However, it also suffers from the problem of equal importance (weight) given to all the items or commodities included in the index.

Price relative: A price index for a given commodity or item that is computed by dividing a current unit price by a base-period unit price and multiplying the result by 100.

Steps of the method to calculate **average price relative** index are summarized as follows:

(i) Select a base year, and then divide the price of each commodity in the current year by the price in the base year, to obtained price relatives.

(ii) Divide the sum of the price relatives of all commodities by the number of commodities used in the calculation of the index.

(iii) Multiply the average value obtained in step (ii) by 100 to express it in percentage.

The formula for computing the index is as follows:

$$\text{Average price relative index } P_{01} = \frac{1}{n}\Sigma\left(\frac{p_1}{p_0}\right)100 \qquad (17\text{-}3)$$

where n = number of commodities included in the calculation of the index.

The average used in computing the index of price relatives could be arithmetic mean or geometric mean. When geometric mean is used for averaging the price relatives, the formula (17-3) becomes

$$\log P_{01} = \frac{1}{n}\Sigma \log\left\{\left(\frac{p_1}{p_0}\right)100\right\} = \frac{1}{n}\Sigma \log P\ ; \quad P = \left(\frac{p_1}{p_0}\right)100$$

Then $$P_{01} = \text{antilog}\left\{\frac{1}{n}\Sigma \log p\right\}$$

Advantages and Limitations of Average Price Relative Index

Advantages: This index has the following advantages over the aggregate price index:

(i) The value of this index is not affected by the units in which prices of commodities are quoted. The price relatives are pure numbers and therefore are independent of the original units in which they are quoted.

(ii) Equal importance is given to each commodity and extreme commodities do not influence the index number.

Limitations: Despite the few advantages mentioned above, this index is not popular on account of the following limitations.

(i) Since it is an unweighted index, therefore each price relative is given equal importance. However in actual practice a few price relatives are more important than others.

(ii) Although arithmetic mean is often used to calculate the average of price relatives, it also has a few biases. The usė of geometric mean is computationally difficult. Other measures of central tendency such as median, mode and and harmonic mean, are almost never used for calculating this index.

(iii) Index of price relatives does not satisfy all criteria such as identity, time reversal, and circular properties, laid down for an ideal index. These criteria will be discussed later in the chapter.

Example 17.3: From the data given below, construct the index of price relatives for the year 2002 taking 2001 as base year using (a) arithmetic mean and (b) geometric mean.

Expenses on	:	*Food*	*Rent*	*Clothing*	*Education*	*Misc.*
Price (Rs), 2001	:	1800	1000	700	400	700
Price (Rs), 2002	:	2000	1200	900	500	1000

Solution: Calculations of Index number using arithmetic mean (A.M.) is shown in Table 17.4

Table 17.4: Calculation of Index Using A.M.

Expenses on	*Price in 2001(p_0)*	*Price in 2000(p_1)*	*Price Relatives* $\frac{p_1}{p_0}\times 100$
Food	1800	2000	111.11
Rent	1000	1200	120.00
Clothing	700	900	128.57
Education	400	500	125.00
Miscellaneous	700	1000	142.86
			627.54

$$\text{Average of price relative index } P_{01} = \frac{1}{n}\Sigma\left(\frac{p_1}{p_0}\right)100$$

$$= \frac{1}{5}(627.54) = 125.508$$

Hence, we conclude that prices of items included in the calculation of index have increased by 25.508% in 2002 as compared to the base year 2001.

(b) Index number using geometric mean (G.M.) is shown in Table 17.5

Table 17.5: Calculations of Index Using G.M.

Expenses on	*Price in 2001* (p_0)	*Price in 2002* (p_2)	*Price Relatives* $P = \frac{p_1}{p_0} \times 100$	*Log P*
Food	1800	2000	111.11	2.0457
Rent	1000	1200	120.00	2.0792
Clothing	700	900	128.57	2.1090
Education	400	500	125.00	2.0969
Miscellaneous	700	1000	142.86	2.1548
				10.4856

$$\text{Average price relative index } P_{01} = \text{antilog}\left\{\frac{1}{n}\Sigma\log p\right\} = \text{antilog}\left\{\frac{1}{5}(10.4856)\right\}$$

$$= \text{antilog } (2.0971) = 125.00$$

Self-Practice Problems 17A

17.1 The following data concern monthly salaries for the different classes of employees within a small factory over a 3-year period.

Employee Class	*Salary per Month* 1998	1999	2000
A	2300	2500	2600
B	1900	2000	2300
C	1700	1700	1800
D	1000	1100	1300

Using 1998 as the base year, calculate the simple aggregate price index for the years 1999 and 2000.

17.2 The following data describe the average salaries (Rs in 1000) for the employees in a company over ten consecutive years.

Year :	1	2	3	4	5
Average salary :	10.9	11.4	12.0	12.7	13.6
Year :	6	7	8	9	10
Average salary :	14.4	15.0	15.5	16.3	17.6

(a) Calculate an index for these average salaries using year 5 as the base year.

(b) Calculate the percentage points change between consecutive years.

17.3 A state Govt. had compiled the information shown below regarding the price of the three essential commodities: wheat, rice, and sugar. From the commodities listed, the corresponding price indicates the average price for that year. Using 1998 as the base year, express the price for the years 2000 to 2002, in terms of unweighted aggregate index.

Commodity	*1998*	*1999*	*2000*	*2001*	*2002*
Wheat	4	6	8	10	12
Rice	16	20	24	30	36
Sugar	8	10	16	20	24

17.4 Following are the prices of commodities in 2003 and 2004. Calculate a price index based on price relatives, using the geometric mean.

Year	*Commodity* A	*B*	*C*	*D*	*E*	*F*
2003	45	60	20	50	85	120
2004	55	70	30	75	90	130

17.5 A textile worker in the city of Mumbai earns Rs 3500 per month. The cost of living index for a particular month is given as 136. Using the following information, find out the amount of money he spent on house rent and clothing.

Group	*Expenditure (Rs)*	*Group Index*
Food	1400	180
Clothing	x	150
House rent	y	100
Food and lighting	560	110
Misc.	630	80

[*Delhi Univ., BCom, 1997*]

17.6 In 1996, for working class people, wheat was selling at an average price of Rs 160 per 10 kg, cloth at Rs 40 per metre, house rent Rs 10,000 per house, and other items at Rs 100 per unit. By 1997 the cost of wheat rose by Rs 40 per 10 kg, house rent by Rs 1500 per house, and other items doubled in price. The working class cost of living index for the year 1997 (with 1996 as base) was 160. By how much did the cloth price rise during the period 1996–97?

17.7 From the following data calculate an index number using family budget method for the year 1996 with 1995 as the base year.

Commodity	*Quantity (in units) in 1995*	*Price (in Rs) per unit*	
		1995	*1996*
A	110	8.00	12.00
B	25	6.00	7.50
C	10	5.00	5.25
D	20	48.00	60.00
E	25	15.00	16.50
F	30	9.00	27.00

[*Karnataka Univ., BCom, 1997*]

17.8 The following table gives the annual income of a teacher and the general index of price during 1990–97. Prepare the index number to show the change in the real income of the teacher and comment on price increase:

Year	*Income*	*Index*
1990	4000	100
1991	4400	130
1992	4800	160
1993	5200	220
1994	5600	270
1995	6000	330
1996	6400	400
1997	6800	490

[*HP Univ., BCom, 1997*]

Hints and Answers

17.1 Simple aggregate price index

$$P_{0,89} = \frac{7300}{6900} \times 100 = 105.8 \text{ for the year 1999}$$

$$P_{0,90} = \frac{8000}{6900} \times 100 = 115.9 \text{ for the year 2000}$$

17.2 (a)

Year :	1	2	3	4	5
Index number :	80.1	83.8	88.2	93.4	100
Year :	6	7	8	9	10
Index number :	105.9	110.3	114.0	119.9	129.4

For example, index for year 1:(10.9 ÷ 13.6)100 = 80.1; year 2: (11.4 ÷ 13.6)100 = 83.8

(b)

Year	*Index number*	*Percentage point change*
1	80.1	—
2	83.8	3.7
3	88.2	4.4
4	93.4	5.2
5	100.0	6.6
6	105.9	5.9
7	110.3	4.4
8	114.0	3.7
9	119.9	5.9
10	129.4	9.5

17.3 Aggregate price

1998	1999	2000	2001	2002
100	133.33	137.78	125	120

17.4

Commodity	$P = \frac{P_1}{P_0} \times 100$	Log P
A	122.22	2.0872
B	116.67	2.0669
C	150.00	2.1761
D	150.00	2.1761
E	105.88	2.0248
F	108.33	2.0348

$$P_{01} = \text{antilog}\left\{\frac{1}{n}\log P\right\} = \text{antilog}\left\{\frac{1}{6}(12.5659)\right\}$$

$$= \text{antilog}\,(2.0948) = 124.4$$

17.5 Let expenditure on clothing be x and on house rent be y. Then as per conditions given, we have

$$3500 = 1400 + x + y + 560 + 630$$

or $\quad x + y = 910 \qquad$ (i)

Multiplying expenditure with group index and equating it to 136, we get

$$136 = \frac{(1400\times180)+(x\times150)+(y\times100)+(500\times110)+(630\times80)}{3500}$$

$$136 = \frac{2,52,000+150x+100y+61,600+50,400}{3500}$$

$$4,76,000 = 2,52,000 + 150x + 100y + 61,600 + 50,400$$

$150x + 100y = 1,12,000 \qquad$ (ii)

Multiplying Eqn. (i) by 150 and subtracting it from (ii), we get

$$50y = 24,500 \text{ or } y = \text{Rs } 490 \text{ (house rent)}$$

Substituting the value of y in Eqn. (*i*): $x + 490 = 910$ or x = Rs 420 (clothing)

17.6 Let the rise in price of cloth be x.

Commodity	Price	Index	Price 1997	Index
Wheat	160	100	200	$\frac{200}{160}\times100 = 125$
Cloth	40	100	x	$\frac{x}{40}\times100 = 2.5x$
House rent	10,000	100	11,500	$\frac{11,500}{10,000}\times100 = 115$
Miscellaneous	100	100	200	$\frac{200}{100}\times100 = 200$
Total				440 + 2.5x

The index for 1997 as given is 160. Therefore, the sum of the index numbers of the four commodities would be 160 × 4 = 640. Thus 440 + 2.5x = 640 or x = 80. Hence the rise in the price of ~~cloth was~~ Rs 40 (80 – 40) per metre.

17.7

Commodity	Quantity Q	p_0	p_1	$P = \frac{p_1}{p_0}\times 100$	PQ
A	100	8	12.00	150	15,000
B	25	6	7.50	125	3,125
C	10	5	5.25	105	1,050
D	20	48	60.00	125	2,500
E	25	15	16.50	110	2,750
F	30	9	27.00	300	9,000
Total	210				33,425

$$\text{Index number} = \frac{\Sigma PQ}{\Sigma Q} = \frac{33,425}{210} = 159.17$$

17.8

Year	Income (Rs)	Index	Real Income (Rs)	Real Income Index
1990	4000	100	$\frac{4000}{100}\times100 = 4000.00$	100.00
1991	4400	130	$\frac{4400}{130}\times100 = 3384.62$	84.62
1992	4800	160	$\frac{4800}{160}\times100 = 3000.00$	75.00
1993	5200	220	$\frac{5200}{220}\times100 = 2363.64$	59.09
1994	5600	270	$\frac{5600}{270}\times100 = 2074.07$	51.85
1995	6000	330	$\frac{\Sigma p_1q_1}{\Sigma p_0q_1}\times100 = 1818.18$	45.45
1996	6400	400	$\frac{6400}{400}\times100 = 1600.00$	40.00
1997	6800	490	$\frac{6800}{490}\times100 = 1387.76$	34.69

17.7 WEIGHTED PRICE INDEXES

While constructing weighted price indexes, rational weights are assigned to all items or commodities in an explicit manner. Such weights indicate the relative importance of items or commodities included in the calculation of an index. The weights used are of two types, *quantity weights* and *value weights*. There are two price indexes that are commonly in use

1. Weighted aggregate price index
2. Weighted average of price relative index

17.7.1 Weighted Aggregate Price Index

In a weighted aggregate price index, each item in the basket of items chosen for calculation of the index is assigned a weight according to its importance. In most cases, the quantity of usage is the best measure of importance. Hence, we should obtain a measure of the quantity of usage for the various items in the group. This explicit weighting allows us to gather more information than just the change in price over a period of time as well as improve the accuracy of the general price level estimate.

Weight is assigned to each item in the basket in various ways and the weighted aggregates are also used in different ways to calculate an index. A few methods (or approaches) to determine weights (value) to be assigned to each item in the basket are as follows:

- Laspeyre's method
- Paasche's method
- Dorbish and Bowley's method
- Fisher's ideal method
- Marshall-Edgeworth's method
- Walsch's method
- Kelly's method

Laspeyre's Weighting Method

This method suggests to treat quantities as constant at *base period* level and are used for weighting price of each item or commodities both in base period and current period. Since this index number depends upon the same base price and quantity, therefore one can directly compare the index of one period with another. The formula for calculating *Laspeyre's price index*, named after the statistician Laspeyre's is given by

Laspeyre's index: A weighted aggregate price index in which the weight for each commodity or item is its base-period quantity.

$$\text{Laspeyre's price index} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$$

where p_1 = prices in the current period
p_0 = prices in the base period
q_0 = quantities consumed in the base period

Advantages and Disadvantages of Laspeyre's Method

Advantages: The main advantage of this method is that it uses only one quantity measure based on the base period and therefore we need not keep record of quantity consumed in each period. Moreover, having used the same base period quantity, we can compare the index of one period with another directly.

Disadvantages: We know that the consumption of commodities decreases with relatively large increases in price and vice versa. Since in this index the fixed quantity weights are determined from the base period usage, it does not adjust such changes in consumption and therefore tends to result in a bias in the value of the composite price index.

Example 17.4: Compute the cost of living index number using Laspeyre's method, from the following information:

Commodity	*Unit Consumption in Base Period*	*Price in Base Period*	*Price in Current Period*
Wheat	200	1.0	1.2
Rice	50	3.0	3.5
Pulses	50	4.0	5.0
Ghee	20	20.0	30.0
Sugar	40	2.5	5.0
Oil	50	10.0	15.0
Fuel	60	2.0	2.5
Clothing	40	15.0	18.0

Solution: Calculation of cost of living index by Laspeyre's method is shown in Table 17.6.

Table 17.6: Laspeyre's Method

Commodity	*Base Period Quantity* (q_0)	*Base Period Price* (p_0)	*Current Price* (p_1)	p_1q_0	p_0q_0
Wheat	200	1.0	1.2	240	200
Rice	50	3.0	3.5	175	150
Pulses	50	4.0	5.0	250	200
Ghee	20	20.0	30.0	600	400
Sugar	40	2.5	5.0	200	100
Oil	50	10.0	15.0	750	500
Fuel	60	2.0	2.5	150	120
Clothing	40	15.0	18.0	720	600
Total	510			3085	2270

$$\text{Cost of living index} = \frac{\Sigma p_1q_0}{\Sigma p_0q_0}\times 100 = \frac{3085}{2270}\times 100 = 135.9$$

Paasche's index: A weighted aggregate price index in which the weight for each commodity or item is its current-period quantity.

Paasche's Weighting Method

In the Paasche's method, the price of each item or commodity is weighted by the quantity in the current period instead of the base year as used in Laspeyre's method. Paasche's formula for calculating the index is given by

$$\text{Paasche price index} = \frac{\Sigma p_1q_1}{\Sigma p_0q_1}$$

where p_1 = prices in current year

p_0 = prices in base year

q_1 = quantities in current year

Advantages and Disadvantages of the Paasche's Method

Advantages: The Paasche's method combines the effects of changes in price and quantity consumption patterns during the current year. It provides a better estimate of changes in the economy than Laspeyre's method. If the prices or quantities of all commodities or items change in the same ratio, then the values of the Laspeyre's and Paasche's indexes will be same.

Disadvantages: This method requires knowledge of the quantities consumed of all commodities in each period. Getting the data on the quantities for each period is either expensive or time-consuming. Moreover, each year the index number for the previous year requires recomputation to reflect the effect of the new quantity weights. Thus, it is difficult to compare indexes of different periods when calculated by the Paasche's method.

Example 17.5: For the following data, calculate the price index number of 1999 with 1998 as the base year, using: (a) Laspeyre's method, and (b) Paasche's method.

Commodity	*1998*		*1999*	
	Price	*Quantity*	*Price*	*Quantity*
A	20	8	40	6
B	50	10	60	5
C	40	15	50	15
D	20	20	20	25

[*Kurukshetra Univ., MBA, 1999*]

Solution: Table 17.7 presents the information necessary for both Laspeyre's and Paasche's methods.